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STARLIKE AND CONVEX MAPPINGS IN SEVERAL COMPLEX VARIABLES

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In this paper, using the Bergman kernel function $K_D(z, \bar{z})$, we give necessary and sufficient conditions that a pseudo-conformal mapping $f(z)$ be starlike or convex in some bounded schlicht domain D for which the kernel function $K_D(z, \bar{z})$ becomes infinitely large when the point $z \in D$ approaches the boundary of D in any way. We also consider starlike and convex mappings from the polydisk or unit hypersphere into C^n.

Generalizing the results obtained by M. S. Robertson [10] using the principle of subordination, T. J. Suffridge has established necessary and sufficient conditions that a function be univalent and map the polydisk or

$$D_p = \left\{z\colon \left[\sum_{j=1}^{n} |z_j|^p\right]^{1/p} < 1,\ p \geqq 1\right\}$$

onto a starlike or convex domain [11].

Similar problems have been considered by T. Matsuno [8] for the hypershere. In this paper we deal with the same problems in terms of the Bergman kernel function $K_D(z, \bar{z})$, and show the results are equivalent to theorems of Suffridge in case of polydisk or hypersphere.

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1. Preliminaries. We consider bounded schlicht domains D in C^n for which the kernel function becomes infinite everywhere on the boundary ∂D, i.e., it is the union of an increasing sequence of strictly pseudo-convex domains

$$D_t = [z\colon \varphi_t(z) \equiv K_D(z, \bar{z}) - t < 0,\ z \in D] \tag{1.1}$$

for some number $t > 0$, where $z = (z_1, \cdots, z_n)'$. (See [3]). First we have

LEMMA 1.1. *If D is a bounded domain, the Bergman kernel function $K_D(z, \bar{z})$ is strictly plurisubharmonic and*

$$1/\omega(D) \leqq K_D(z, \bar{z}) \leqq 1/\pi^n(l(z))^{2n}\,, \tag{1.2}$$

where $l(z) = \min_{\tau \in \partial D} \rho(\tau, z)$, $\rho(\tau, z) = \max_j\{|\tau_j - z_j|, j = 1, \cdots, n\}$ and $\omega(D)$ signifies the euclidean volume of D.