PACIFIC JOURNAL OF MATHEMATICS
Vol. 44, No. 2, 1973

BOUNDED ENTIRE SOLUTIONS OF ELLIPTIC EQUATIONS

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Let

$$Lu = \sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b_i(x)\frac{\partial u}{\partial x_i} \,. \tag{1.1}$$

Consider the equation

$$Lu(x) = f(x) \,. \tag{1.2}$$

It is shown, under some general conditions on the coefficients of L, that if $f(x)$ is locally Hölder continuous and

$$f(x) = O(|x|^{-2-\mu}) \quad \text{as} \quad |x| \longrightarrow \infty \qquad (\mu > 0) \tag{1.3}$$

then there exists a bounded solution of (1.2) in R^n when $n \geqq 3$. If $n = 2$ then bounded entire solutions may not exist, but there exists a nonnegative solution of (1.2) in R^2 which is bounded above by $O(\log |x|)$. An application of these results to the Cauchy problem is given in the final section of the paper.

If in (1.3) $\mu = 0$ then already the equation $\Delta u = f$ ($n \geqq 3$) may not have an entire bounded solution; an example is given by Meyers and Serrin [4].

2. Existence of a bouned solution. We shall need the following conditions:

$$\sum_{i,j=l}^{n} a_{ij}(x)\xi_j\xi_i > 0 \quad \mathit{if} \ x \in R^n, \xi \in R^n, \xi \neq 0 \,, \tag{2.1}$$

(2.2) $a_{ij}(x)$, $b_i(x)$ are bounded, locally Hölder continuous in R^n $(1 \leqq i, j \leqq n)$,

$$\text{For some } \delta > 0, R > 0, 0 < \rho < 1 \,, \tag{2.3}$$

$$(2 + \delta)\,|x|^{-2} \sum_{i,i=i}^{n} a_{ij}(x)x_i x_j \leqq \rho \sum_{i=1}^{n} a_{ii}(x) + \sum_{i=1}^{n} x_i b_i(x) \ \text{ if } |x| > R \,,$$

$$\sum_{i=1}^{n} a_{ii}(x) \geqq \gamma > 0 \text{ for all } x \in R^n \qquad (\gamma \text{ constant}) \,. \tag{2.4}$$

Notice that (2.1) and (2.4) both follow from the condition of uniform ellipticity

$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geqq \gamma_0\,|\xi|^2 \text{ for all } x \in R^n,\ \xi \in R^n \qquad (\gamma_0 \text{ positive constant}) \,. \tag{2.5}$$