PACIFIC JOURNAL OF MATHEMATICS
Vol. 45, No. 1, 1973

VARIETIES OF IMPLICATIVE SEMI-LATTICES II

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This paper is concerned with a process of coordinatization of the lattice of varieties of implicative semilattices. Equational descriptions of some elements in each coordinate class, and a complete equational description of one coordinate class are given.

1. Introduction.

This paper is a continuation of [8]. Familiarity with [8] and [6] is assumed. After stating some of the consequences of the local finiteness of the variety of implicative semilattices, we describe a system for partitioning the lattice of varieties of implicative semi-lattices into coordinate intervals, and give some results that can be obtained from a study of this coordinatization. Finally, we give equational descriptions for the largest and smallest varieties in each coordinate class, the covers of the smallest variety in each coordinate class and a complete equational description of the coordinate class 4,2.

Recall that an implicative semi-lattice is subdirectly irreducible if and only if it has a single dual atom. In accordance with the usage of [8], this dual atom will be denoted by u. If in a subdirectly irreducible implicative semi-lattice, the dual atom is deleted, the remaining structure is both a subalgebra and a homomorphic image of the original. Thus every subdirectly irreducible implicative semi-lattice may be thought of as obtained by appending a single dual atom to some already given implicative semi-lattice. If L is an implicative semilattice, the subdirectly irreducible implicative semi-lattice obtained in this manner will be denoted by $\hat{L}$.

2. Local finiteness.

The following theorem was proven first by A. Diego [2] in a slightly different context. McKay [4] extended the result to implicative semi-lattices. We present a much simpler proof here.

THEOREM 2.1. *The variety of implicative semi-lattices is locally finite.*

Proof. Let F_n denote the free implicative semi-lattice on n generators. The proof proceeds by induction. F_1 has two elements. Assume that F_n is finite. $F_{n+1} \leqq_s \prod \hat{L}_i$, where each $\hat{L}_i$ is $n+1$ generated. Hence each L_i is n generated. It follows from the induction assumption that there are only a finite number of distinct L_i each