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MULTIPLIERS OF TYPE (p, p) AND MULTIPLIERS OF THE GROUP L_p-ALGEBRAS

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Let G be a locally compact group with left Haar measure λ and suppose $1 \leqq p < \infty$. The purpose of this paper is to exhibit an isometric isomorphism ω of the Banach algebra M_p of all right multipliers on $L_p = L_p(G, \lambda)$ into the normed algebra m_p of all right multipliers on the group L_p-algebra L_p^t. When G is either commutative or compact, ω is surjective.

A function $f \in L_p$ is said to be *p-temperate* if

(1) $\quad h*f(x) = \int_G f(t)h(t^{-1}x)d\lambda(t)$ exists for λ-almost all $x \in G$ whenever h is in L_p;

(2) $\quad h*f$ is in L_p for all $h \in L_p$;

(3) $$\sup\{\|h*f\|_p : h \in L_p, \|h\|_p \leqq 1\} < \infty .$$

It was shown in [6], Theorem 1, that $f \in L_p$ is p-temperate if

(4) $$\sup\{\|h*f\|_p : h \in C_{\infty}, \|h\|_p \leqq 1\} < \infty$$

where C_{00} denotes the set of all continuous complex-valued functions on G with compact support. The set of all p-temperate functions will be written as L_p^t. Each function $f \in C_{00}$ is in L_p^t and so L_p^t comprises a dense subspace of L_p. For $f \in L_p^t$, the number given by either (3) or (4) will be written as $\|f\|_p^t$. The function $\| \; \|_p^t$ so defined is a norm under which L_p^t is a normed algebra. This normed algebra will be referred to as the *group L_p-algebra*.

By a right multiplier on L_p^t will be meant a bounded linear operator T on L_p^t such that

(5) $$T(f*g) = f*T(g) \qquad \text{for all } f \text{ and } g \text{ in } L_p^t .$$

The set of all such T, which constitutes a normed algebra under the usual operator norm, will be written as $\mathfrak{m}_p$. Write $\mathfrak{B}p$ for the Banach algebra of all bounded linear operators on L_p. An operator $T \in \mathfrak{B}p$ is said to be a right multiplier of type (p, p) (see [3]) if

(6) $$T({}_xf) = {}_xT(f) \qquad \text{for all } f \in L_p$$

where ${}_xh(y) = h(xy)$ for each function h on G. The set of all such T will be written as M_p. It is a complete sub-algebra of $\mathfrak{B}_p$.

The group L_p-algebra was utilized in [6] to study a related algebra A_p, of which the Banach algebra of left multipliers was found