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FACTORED CODIMENSION ONE CELLS IN EUCLIDEAN n-SPACE

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Seebeck has proved that if the m-cell C in Euclidean n-space E^n factors k times, where $m \leqq n-2$ and $n \geqq 5$, then every embedding of a compact k-dimensional polyhedron in C is tame relative to E^n. In this note we prove the analogous result for the case $m+1=n \geqq 5$ and $n-k \geqq 3$. In addition we show that if C factors 1 time, then each $(n-3)$-dimensional polyhedron properly embedded in C can be homeomorphically approximated by polyhedra in C that are tame relative to E^n.

Following Seebeck [8] we say that an m-cell C in E^n *factors k times* if for some homeomorphism h of E^n onto itself and some $(m-k)$-cell B in E^{n-k}, $h(C) = B \times I^k$, where I^k denotes the k-fold product of the interval I naturally embedded in E^k and where

$$B \times I^k \subset E^{n-k} \times E^k = E^n$$

is the product embedding.

In another paper [6] the author has studied results comparable to Seebeck's for factored cells in E^4, but the techniques employed here differ slightly from those used in [6] and [8]. The main result generalizes work of Bryant [2], and the final section here expands on his methods to obtain a strong conclusion about tameness of all subpolyhedra in certain factored cells.

1. **Definitions and Notation.** For any point p in a metric space S and any positive number δ, $N_\delta(p)$ denotes the set of points in S whose distance from p is less than δ.

The symbol Δ^2 denotes a 2-simplex fixed throughout this paper, $\partial\Delta^2$ its boundary, and Int Δ^2 its interior.

Let A denote a subset of a metric space X and p a limit point of A. We say that A is *locally simply connected at* p, written 1-LC at p, if for each $\varepsilon > 0$ there is a $\delta > 0$ such that each map of $\partial\Delta^2$ into $A \cap N_\delta(p)$ can be extended to a map of Δ^2 into $A \cap N_\varepsilon(p)$. Furthermore, we say that A is *uniformly locally simply connected*, written 1-ULC, if for each $\varepsilon > 0$ there is a $\delta > 0$ such that each map of $\partial\Delta^2$ into a δ-subset of A can be extended to a map of Δ^2 into an ε-subset of A. Similarly, we say that A is *locally simply connected in X at p*, written 1-LC in X at p, if for each $\varepsilon > 0$ there is a $\delta > 0$ such that each map of $\partial\Delta^2$ into $A \cap N_\delta(p)$ extends to a map of Δ^2 into $N_\varepsilon(p)$, and we say that A is *uniformly locally simply connected in* X (1-ULC in X) if the corresponding uniform property is satisfied.