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ON EXTENDING ISOTOPIES

WILLIAM H. CUTLER

Let K be a locally compact metric space. An *isotopy* on K is a continuous family of homeomorphisms $h_t: K \to K$ for $t \in I$ such that $h_0 = \mathrm{id}$. Let $\mathscr{I}(K)$ denote the space of isotopies of K with $C-0$ topology. *Conjecture*: Let X be metric and Y a closed subset of X. Then every map $f: Y \to \mathscr{I}(K)$ can be continuously extended to X. The conjecture is proved for the following cases: (1)K is a 1-complex, (2)K is compact and X is finite-dimensional, (3)K is compact, Y is compact and finite-dimensional, and X is separable, and (4)Y is of type 1 in a compact space X. Y is of *type* 1 *in* X if the closure of the set of points of X which do not have a unique closest point in Y does not intersect Y.

1. **Introduction.** Let K be a topological space. An *isotopy on* K is a continuous family, for $t \in I$, of homeomorphisms $h_t: K \longrightarrow K$ such that $h_0 = \mathrm{id}$. The isotopy is *invertible* if $g_t = (h_t)^{-1}$ is also an isotopy (i.e., is continuous in t). Invertible isotopies can be thought of as level-preserving homeomorphisms of $K \times I$ onto itself which are the identity on $K \times \{0\}$. Throughout the paper, K will be locally compact and metric, so all isotopies on K will automatically be invertible. We will denote the space of isotopies on K with $C - 0$ topology by $\mathscr{I}(K)$. We will discuss the following:

Conjecture. Let K be locally compact and metric. Let X be metric and let Y be a closed subset of X. Then every map $\varphi: Y \longrightarrow \mathscr{I}(K)$ can be continuously extended to X.

It should be noted that this conjecture is equivalent to saying that $\mathscr{I}(K)$ is an absolute retract for metric spaces. The following theorem states the conjecture for several special cases:

THEOREM. *The conjecture is true in the following cases:*

(1) *K is a one-dimensional simplicial complex (for example, R^1, S^1, or I)*

(2) *K is compact, X is finite-dimensional*

(3) *K is compact, Y is finite-dimensional and compact, and X is separable.*

(4) *X is compact and Y is of type* 1 *in X.*

The last case is an easy result, the definition of type being as follows: Let Y be a closed subset of X. Define $H(Y) = \{x \in X \mid x$ does not have a unique closest point in $Y\}$. Let $Y_1 = \overline{H(Y)} \cap Y$, $Y_n = Y_{n-1} \cap \overline{H(Y_{n-1})}$ for $n = 2, 3, \cdots$.