PACIFIC JOURNAL OF MATHEMATICS
Vol. 47, No. 1, 1973

# THE WEAK ENVELOPE OF HOLOMORPHY FOR ALGEBRAS OF HOLOMORPHIC FUNCTIONS

ALAN T. HUCKLEBERRY

**The object of this paper is to study analytic continuation of algebras of functions holomorphic on complex spaces of dimension greater than 1. Classically this has been done by putting complex structure on the maximal spectrum of the algebra so that the spectrum is a Stein space with respect to the induced algebra of holomorphic functions. Grauert has given non-pathological examples where this is not possible. In the present paper the axioms of a Stein space have been weakened and the weak envelope of holomorphy has been constructed for a certain type of algebra. In particular, if the algebra $A$ separates points and gives local coordinates on a complex space $X$ then the weak envelope of holomorphy for the pair, $(X, A)$ is obtained.**

1. Introduction. In this paper a complex space, unless otherwise stated, will mean a normal, connected, reduced complex space. We will let $H(X)$ denote the algebra of functions holomorphic on a complex space $X$. A complex space $E$ is said to be the envelope of holomorphy of a complex space $X$ if the following conditions are satisfied:

(1) There is a holomorphic mapping $\tau: X \to E$ such that $\tau(X)$ is open in $E$.

(2) The map $\tau^*: H(E) \to H(X)$ is an algebra isomorphism.

(3) The complex space $E$ is a Stein space.

It is known that if $X$ has an envelope of holomorphy, $E$, and $\tau': X \to E'$ satisfies (1) and (2) above then there is a biholomorphic mapping $\varphi: E' \to E$ such that $\tau = \varphi \circ \tau'$. Moreover, the spectrum of $H(X)$, $S(H(X))$, has the structure of a complex space such that it is biholomorphically equivalent to $E$ in a natural way [2].

Grauert has provided an example of a complex manifold $X$ with $H(X)$ containing local coordinates and separating points, but $S(H(X))$ contains a point no neighborhood of which has the structure of an analytic variety [1]. Thus, in order to investigate the envelope of holomorphy problem, it makes sense to either modify the notion of a complex space, as Grauert has suggested [1], or to weaken the constraints on an envelope of holomorphy. In this paper we have taken the latter route. We have weakened the restrictions (1), (2), and (3) above, while preserving the maximality property. Our results, which apply to a class of algebras which includes many algebras which are not of the form $H(X)$, can be described as follows.