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ISOMORPHIC CLASSES OF THE SPACES $C_\sigma(S)$

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Jerison introduced the Banach spaces $C_\sigma(S)$ of continuous real or complex-valued odd functions with respect to an involutory homeomorphism $\sigma: S \to S$ of the compact Hausdorff space S. It has been conjectured that any Banach space of the type $C_\sigma(S)$ is isomorphic to a Banach space of all continuous functions on some compact Hausdorff space. This conjecture is shown to be true if either (1) S is a Cartesian product of compact metric spaces or (2) S is a linearly ordered compact Hausdorff space and σ has at most one fixed point.

Introduction. Let S always denote a compact Hausdorff space. $C(S)$ well denote the Banach space of real or complex-valued continuous functions on S equipped with the supremum norm. A homeomorphism $\sigma: S \to S$ is involutory if $\sigma(\sigma(s)) = s$ for each $s \in S$. Jerison [2] introduced the Banach space $C_\sigma(S) = \{f \in C(S): f(\sigma(s)) = -f(s)\}$ of odd functions with respect to an involutory homeomorphism $\sigma: S \to S$. If X and Y are Banach spaces then X is *isomorphic* (*isometric*) to Y, and we will write $X \sim Y$ ($X \approx Y$), if there is a bounded (norm preserving) one-to-one linear operator from X onto Y.

A special case of a conjecture due to A. Pełczyński [8] is as follows: for any Banach space $C_\sigma(S)$ there is a compact Hausdorff space T with $C_\sigma(S) \sim C(T)$. In this paper we prove this conjecture when S is either a Cartesian product of compact metric spaces or a linearly ordered compact Hausdorff space (in the second case we assume σ has at most one fixed point). The results and techniques of this paper generalize, and provide shorter proofs of, some results of Samuel [11].

1. Linearly ordered spaces.

A topological space A is a *linearly ordered topological space* if the topology on A is the order topology ([4], page 57) arising from some linear ordering on the set A. Examples of linearly ordered spaces are the closed interval [0,1], every space of ordinal numbers, every totally disconnected compact metric space ([5], Corollary 2a), and every compact subset of a linearly ordered space.

THEOREM 1. *Let S be an infinite linearly ordered compact Hausdorff space. If σ is an involutory homeomorphism on S with at most one fixed point, then $C_\sigma(S) \sim C(T)$ for some compact Hausdorff space T.*