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COMPOSANTS OF HAUSDORFF INDECOMPOSABLE CONTINUA; A MAPPING APPROACH

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"Continuum" denotes a compact connected Hausdorff space. The principal result is that every indecomposable continuum can be mapped onto Knaster's example D of a chainable indecomposable continuum with one endpoint. This result is then used to conclude that those indecomposable continua each of whose proper subcontinua is decomposable, those which are homeomorphic with each of their nondegenerate subcontinua, and those such that each two points in the same composant can be joined by a continuum which cannot be mapped onto D, have at least c composants. It is also shown that generalized arcwise connected continua are decomposable.

The author [1] and [2], among others, has raised the question of how many composants an indecomposable continuum must have. The technique applied to prove that metric indecomposable continua have uncountably many depends upon the second countability of the complement of a point. (See, for example, [5, p. 140].) Other arguments can generalize this; for example, H. Cook has pointed out in conversation that if an indecomposable continuum has two composants, and is first countable at a point of each, then it has uncountably many composants. This can be generalized to include the case of a continuum with two composants, each of which contains a compact G_δ subset. S. Mazurkiewicz [7] has shown that a metric indecomposable continuum has c composants, sharpening slightly the result that it has uncountably many. M. E. Rudin [10] has shown that if the continuum hypothesis holds, then it is not true that every indecomposable continuum has c composants.

J. W. Rogers, Jr., [9] has shown that every metric indecomposable continuum can be mapped onto the continuum D mentioned above. (See [5, p. 332] or [6, p. 206] for a picture.) We follow Rogers here in a representing D as an inverse limit of arcs $[0, 1] = I$, indexed by the positive integers, where the bonding map between successive terms is always h, where $h(t) = 2t$ for $t \leqq 1/2$ and $h(t) = 2 - 2t$ for $t \geqq 1/2$. Throughout what follows, let I denote $[0, 1]$; h, this function; and D, the inverse limit of this system. This work extends Rogers' result to the nonmetric case; also, the argument here is simpler than Rogers'. This result is then applied to obtain a partial answer to the composant question in certain cases. This work also generalizes work of G. R. Gordh, Jr., presented at the University of Oklahoma Conference on General Topology in March, 1972, [3], and answers in the negative