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POSITIVE-DEFINITE DISTRIBUTIONS AND INTERTWINING OPERATORS

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An example is given of a positive-definite measure μ on the group SL(2, $\boldsymbol{R}$) which is extremal in the cone of positive-definite measures, but the corresponding unitary representation L^μ is *reducible*. By considering positive-definite *distributions* this anomaly disappears, and for an arbitrary Lie group G and positive-definite distribution μ on G a bijection is established between positive-definite distributions on G bounded by μ and positive-definite intertwining operators for the representation L^μ. As an application, cyclic vectors for L^μ are obtained by a simple explicit construction.

Introduction. The use of positive-definiteness as a tool in abstract harmonic analysis has a long history, the most striking early instance being the Gelfand-Raikov proof via positive-definite functions of the completeness of the set of irreducible unitary representations of a locally compact group [5]. More recently, it was observed by R. J. Blattner [1] that the systematic use of positive-definite *measures* gives very simple proofs of the basic properties of induced representations, and the cone of positive-definite measures on a group was subsequently studied by Effros and Hahn [4].

The purpose of this paper is two-fold. First, we give an example to show that positive-definite measures do not suffice for the study of intertwining operators and irreducibility of induced representations, despite the claim to the contrary in [4]. Specifically, we exhibit a positive-definite measure μ on $G = \mathrm{SL}(2, \boldsymbol{R})$ such that μ lies on an extremal ray in the cone of positive-definite measures on G, but the associated unitary representation L^μ is *reducible*, contradicting Lemma 4.16 of [4].

Our second aim is to show that when G is any Lie group, then the correspondence between intertwining operators and positive functionals on G asserted by Effros and Hahn does hold, provided one deals throughout with positive-definite *distributions* instead of just measures. The essential point is the validity of the Schwartz Kernel Theorem for the space $C_0^\infty(G)$, together with a result of Bruhat [3] about distributions on $G \times G$, invariant under the diagonal action of G. Using this correspondence, we obtain cyclic vectors for representations defined by positive-definite distributions, using a modification of the construction in [7]. (The proof of cyclicity given in [7] is invalid, since it assumes the existence of a measure on G corresponding to