PACIFIC JOURNAL OF MATHEMATICS
Vol. 48, No. 2, 1973

THE PICARD THEOREM FOR MULTIANALYTIC FUNCTIONS

P. KRAJKIEWICZ

The class of multianalytic functions are defined. For this class the notions of essential and nonessential isolated singularities and of exceptional values are introduced. It is then shown that a multianalytic function has at most one exceptional value at an essential isolated singularity.

Suppose that G is an arbitrary nonempty region of the finite complex plane Γ and that $n \geqq 0$ is a nonnegative integer. A function $f: G \rightarrow \Gamma$ is said to be $(n+1)$-analytic on G or simply polyanalytic on G if and only if there exist $(n+1) \geqq 1$ functions f_k analytic on G for $k = 0, 1, \cdots, n$ such that

$$f(z) = \sum_{k=0}^{n} \bar{z}^k f_k(z) , \tag{1}$$

for all z in G, where $\bar{z}$ denotes the complex conjugate of z. A function f is said to be $(n+1)$-entire or simply polyentire if and only if this function f is $(n+1)$-analytic on Γ. A function f is said to be bianalytic on G if and only if this function f is $(n+1)$-analytic on G with $n = 1$. Also a function f is termed bientire if and only if this function f is bianalytic on Γ.

Now let $f: G \rightarrow \Gamma$ be a function which is polyanalytic on G and suppose that the function f is represented on G by equation (1). It is not hard to be persuaded that the functions f_k in equation (1) are uniquely determined on G by the function f. With this observation in mind the following definitions are not ambiguous. Let z_0 be an arbitrary complex number, finite or infinite. Then the point z_0 is said to be an isolated singularity of f if and only if there is some neighborhood N of z_0 such that $N - \{z_0\} \subseteqq G$. The point z_0 is called an essential isolated singularity of f or simply an essential singularity of f if and only if the point z_0 is an isolated singularity of at least one of the functions f_k.

In [1], M. B. Balk derived the small Picard theorem for bientire functions by appealing to Picard's big theorem for analytic functions and the theory of quasi-normal families of analytic functions [6, p. 66]. Then in [2], the same author derived the big Picard theorem for bientire functions by applying similar methods. Later in [3], the same author succeeded in establishing Picard's big theorem for the larger class of polyentire functions by utilizing a theorem of Saxer [7], which generalizes the classical Schottky theorem. Finally in [4], a