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ON EXACT LOCALIZATION

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In this paper we consider certain aspects of exact localization (idempotent kernel functors having Property (T) in the language we shall be employing). The major result is that for commutative noetherian rings, every idempotent kernel functor has Property (T) if and only if the Krull dimension of the ring is less than or equal to 1.

1. Preliminaries. The terminology and notation in this paper are that of Goldman [1], with which familiarity is assumed. In particular, if Λ is a ring we denote by $K(\Lambda)$ (respectively $I(\Lambda)$) the set of kernel functors (respectively idempotent kernel functors on the category of left Λ-modules) belonging to Λ. If $\sigma \in K(\Lambda)$, we denote by $\mathscr{T}_\sigma$ the associated filter of left ideals; i.e., $\mathscr{T}_\sigma$ is the set of left ideals $\mathfrak{A}$ of Λ such that $\Lambda/\mathfrak{A}$ is σ-torsion, and such an $\mathfrak{A}$ is called a *σ-open* left ideal. Finally by the term "module" we mean a left module over the ring in question.

Our primary concern will be with kernel functors satisfying any of the conditions of Theorem 4.3 of [1], which we restate for easy reference.

THEOREM 1.1. *For any $\sigma \in I(\Lambda)$, the following conditions are equivalent*:

(i) $Q_\sigma(M) \approx Q_\sigma(\Lambda) \otimes_\Lambda M$ *for every module* M;

(ii) $Q_\sigma(\Lambda)i(\mathfrak{A}) = Q_\sigma(\Lambda)$ *for every* $\mathfrak{A} \in \mathscr{T}_\sigma$, *where* i *is the canonical map* $\Lambda \rightarrow Q_\sigma(\Lambda)$;

(iii) *Every* $Q_\sigma(\Lambda)$*-module is faithfully* σ*-injective as a* Λ*-module*; i.e., *given a* $Q_\sigma(\Lambda)$*-module* X, Λ*-modules* $B \subseteqq A$ *with* $\sigma(A/B) = A/B$ *and a* Λ*-homomorphism* $f: B \rightarrow X$, *there is a unique* Λ*-homomorphsim* g: $A \rightarrow X$ *extending* f;

(iv) *Every* $Q_\sigma(\Lambda)$*-module is* σ*-torsion-free as a* Λ*-module*;

(v) *The functor* Q_σ *is right exact and commutes with direct sums.*

An idempotent kernel functor satisfying any of the above conditions is said to have *Property* (T). Each of the conditions in (v) above has a useful equivalent (Theorems 4.3 and 4.4 of [1]) which we also list.

THEOREM 1.2. *For* $\sigma \in I(\Lambda)$, *the following are equivalent*:

(i) Q_σ *is a right exact functor*;

(ii) *If* $\mathfrak{A} \in \mathscr{T}_\sigma$, *if* $M \xrightarrow{\pi} M''$ *is an epimorphism of* σ*-torsion-free modules, and if* $f: \mathfrak{A} \rightarrow M''$ *is a homomorphism, then there exists* $\mathfrak{B} \in$