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CHARACTERIZATIONS OF λ CONNECTED PLANE CONTINUA

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A continuum M is said to be λ connected if any two of its points can be joined by a hereditarily decomposable continuum in M. Here we characterize λ connected plane continua in terms of Jones' functions K and L.

A nondegenerate metric space that is both compact and connected is called a *continuum*. A continuum M is said to be *aposyndetic at a point p of M with respect to a point q of M* if there exists an open set U and a continuum H in M such that $p \in U \subset H \subset M - \{q\}$.

In [1], F. Burton Jones defines the functions K and L on a continuum M into the set of subsets of M as follows:

For each point x of M, the set $K(x)$ ($L(x)$) consists of all points y of M such that M is not aposyndetic at x (y) with respect to y (x).

Note that for each point x of M, the set $L(x)$ is connected and closed in M [1, Th. 3]. For any point x of M, the set $K(x)$ is closed [1, Th. 2] but may fail to be connected [2, Ex. 4], [3].

Suppose that M is a plane continuum. In this paper it is proved that the following three statements are equivalent.

I. M is λ connected.

II. For each point x of M, the set $K(x)$ does not contain an indecomposable continuum.

III. For each point x of M, every continuum in $L(x)$ is decomposable.

Throughout this paper E^2 is the Euclidean plane. For a given set S in E^2, we denote the closure and the boundary of S relative to E^2 by $\mathrm{Cl}\, S$ and $\mathrm{Bd}\, S$ respectively.

DEFINITION. Let M be a continuum in E^2. A subcontinuum L of M is said to be a *link* in M if L is either the boundary of a complementary domain of M or the limit of a convergent sequence of complementary domains of M.

It is known that a plane continuum is λ connected if and only if each of its links is hereditarily decomposable [5, Th. 2].

THEOREM 1. *Suppose that a continuum M in E^2 contains an indecomposable continuum I, that disjoint circular regions V and Z in E^2 meet I, that a point x belongs to $M - \mathrm{Cl}\,(V \cup Z)$, and that ε is a positive real number. Then there exist continua H and F in I, arc-segments R and T in V, and a point y of $I \cap Z$ such that* (1)