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MULTIPLIERS AND THE GROUP L_p-ALGEBRAS

JOHN GRIFFIN AND KELLY MCKENNON

Let G be a locally compact group, p a number in $[1, \infty[$, and L_p the usual L_p-space with respect to left Haar measure on G. The space L_p^t consists of those functions f in L_p^t such that $g*f$ is well-defined and in L_p for each g in L_p. Since each function in L_p^t may be identified with a linear operator on L_p which, as it turns out, is bounded; the operator norm may be super-imposed on L_p^t and, under this norm $\| \|_p^t$, L_p^t is a normed algebra. The family of right multipliers (i.e., bounded linear operators which commute with left multiplication operators) on any normed algebra A will be written as $\mathfrak{m}_r(A)$ and the family of left multipliers as $\mathfrak{m}_l(A)$. The family of all bounded linear operators on L_p which commute with left translations will be written as $\mathfrak{M}_p$.

It was shown in a previous issue of this journal that the Banach algebra $\mathfrak{M}_p$ is linearly isomorphic to the normed algebra $\mathfrak{m}_r(L_p^t)$, whenever the group G is either Abelian or compact. This fact is shown, in the present paper, to hold for general locally compact G. The norm $\| \|_p^t$ is defective in that, unless $p = 1$, $(L_p^t, \| \|_p^t)$ is never complete.

An attempt will be made in the sequel to supply this deficiency by the introduction of a second norm $|\!|\!| \;|\!|\!|_p^t$ on L_p^t under which L_p^t is always a Banach algebra. It will be seen that, for $p=2$ (and of course for $p=1$), the Banach algebra $\mathfrak{m}_r(L_p^t, |\!|\!| \;|\!|\!|_p^t)$ is linearly isometric to $\mathfrak{M}_p$.

Let G be a fixed, but arbitrary, locally compact topological group with left Haar measure λ. Write C_{00} for the family of continuous, complex-valued functions on G with compact support.

Let p be a fixed, but arbitrary, number in $[1, \infty[$ and write $\| \|_p$ for the norm on the Banach space $L_p = L_p(G, \lambda)$. The group L_p-algebra L_p^t is the set

$$\{f \in L_p : g*f \in L_p \quad \text{for all} \quad g \in L_p\} .$$

A function $f \in L_p$ is said to be p-tempered and, as shown in [3], the number

$$(1) \qquad \|f\|_p^t = \sup\{\|g*f\|_p : g \in C_{00}\ \|g\|_p \leqq 1\}$$

is finite. Conversely, if $\|f\|_p^t$ is finite for some $f \in L_p$, then—as proved in [3]—f is p-tempered and there exists precisely one operator W_f in $\mathfrak{M}_p$ such that

$$\|W_f\| = \|f\|_p^t \quad \text{and} \quad W_f(g) = g*f$$