PACIFIC JOURNAL OF MATHEMATICS
Vol. 51, No. 1, 1974

A CHARACTERIZATION OF THE TOPOLOGY OF COMPACT CONVERGENCE ON $C(X)$

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The function space of all continuous real-valued functions on a realcompact topological space X is denoted by $C(X)$. It is shown that a topology τ on $C(X)$ is a topology of uniform convergence on a collection of compact subsets of X if and only if (*) $C_\tau(X)$ is a locally m-convex algebra and a topological vector lattice. Thus, the topology of compact convergence on $C(X)$ is characterized as the finest topology satisfying (*). It is also established that if $C_\tau(X)$ is an A-convex algebra (a generalization of locally m-convex) and a topological vector lattice, then each closed (algebra) ideal in $C_\tau(X)$ consists of all functions vanishing on a fixed subset of X. Some consequences for convergence structures are investigated.

Introduction. Throughout this paper, X will denote a realcompact topological space and $C(X)$ the algebra and lattice of all real-valued continuous functions on X under the pointwise defined operations. After preliminary remarks in § 1, we describe (Theorem 1) closed (algebra) ideals in $C(X)$ endowed with a topology τ making $C_\tau(X)$ an A-convex algebra (a generalization of locally m-convex introduced in [4]) and a topological vector lattice. As a corollary, we state sufficient conditions for τ so that an ideal in $C_\tau(X)$ is closed if and only if it consists of all functions vanishing on a subset of X. Then, in Theorem 3, we characterize topologies on $C(X)$, which are topologies of uniform convergence on a collection of compact subsets of X. In particular, the corollary of Theorem 3 provides a characterization of the topology of compact convergence on $C(X)$. We conclude the note (§ 3), by discussing generalizations applicable to convergence structures on $C(X)$.

1. Definitions and preliminary results.

Since our major concern is the algebra $C(X)$, we restrict our definitions to commutative algebras over the reals.

DEFINITION 1. Given a commutative R-algebra $\mathscr{A}$, an absolutely convex subset $S \subset \mathscr{A}$ is said to be *m-convex* (respectively, *A-convex*) if $S \cdot S = \{fg: f, g \in S\}$ is contained in S (respectively, $fS = \{fg: g \in S\}$ is absorbed by S for each f in S). Now $(\mathscr{A}, \tau)$, the algebra $\mathscr{A}$ together with a convergence structure τ (see [1]) is said to be an *m-convex* (respectively, *A-convex*) *convergence algebra* if τ is a convergence vector space structure (see [1]) and for every filter θ con-