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THE NON ABSOLUTE NÖRLUND SUMMABILITY OF FOURIER SERIES

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The paper is devoted partly to the study of non-absolute Nörlund summability of Fourier series of $\varphi(t)$ under the condition $\varphi(t)\chi(t) \in AC[0, \pi]$ for suitable $\chi(t)$. The other aspect is to determine the order of variation of the Harmonic mean of the Fourier series whenever $\varphi(t) \log k/t \in BV[0, \pi]$.

1. Let L denote the class of all real functions f with period 2π and integrable in the sense of Lebesgue over $(-\pi, \pi)$ and let the Fourier series of $f \in L$ be given by

$$\sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt) = \sum_{n=1}^{\infty} A_n(t) ,$$

assuming, as we may, the constant term to be zero.

We write

$$\phi(t) = \frac{1}{2}\{f(x+t) + f(x-t)\}$$

$$g(n, t) = \int_0^t \frac{\cos nu}{\chi(u)} du$$

$$h(n, t) = \int_t^{\pi} \frac{\cos nu}{\chi(u)} du .$$

Let $\{p_n\}$ be a sequence of constants such that $P_n = \sum_{v=0}^{n} p_v \neq 0$ $(n \geqq 0)$ and $P_{-1} = p_{-1} = 0$. For the definition of absolute Nörlund or (N, p) method, see, for example, Pati [9]. When $\sum_{n=0}^{\infty} a_n$ is absolutely (N, p) summable, we shall write, for brevity, $\sum_{n=0}^{\infty} a_n \in |N, p|$.

We define the sequence of constants $\{c_n\}$ formally by $(\sum_{n=0}^{\infty} p_n x^n)^{-1} = \sum_{n=0}^{\infty} c_n x^n$, $c_{-1} = 0$.

2. One of the objects of this paper is to study the non-absolute (N, p) summability factors of Fourier series and generalize the following outstanding result of Pati in Theorems 1–2. Besides, the proof of Theorems 1–2 are short and simple and avoids the direct technique of Pati which is somewhat long and complicated.

If we write

$$G = \left\{f : f \in L,\ \varphi(t) \log k/t \in AC[0, \pi] \text{ and } \sum_{n=1}^{\infty} A_n(x) \notin \left|N, \frac{1}{n+1}\right|\right\}$$

then Pati's theorem is in the following form: