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PHRAGMÉN-LINDELÖF TYPE THEOREMS FOR A SYSTEM OF NONHOMOGENEOUS EQUATIONS

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Hyperbolic systems of N equations are considered

(1) $\partial u/\partial t = P(D)u + f(x, t)$, **with** u **in** R^N **and** (x, t) **in** $R^n \times R^1$,

where $D = i\partial/\partial x$. **For suitable source functions** $f(x, t)$ **there are solutions satisfying the boundedness condition**

(2) $|u(x, t)| \leqq C \exp\{a|x|^\gamma + b|t|^\theta\}$, $0 \leqq \theta < 1$, $0 \leqq \gamma < p$,

where p **is the conjugate of** $2p_0$, **with** p_0 **the reduced order of the matrix** $P(\xi)$. **Furthermore, the solutions are polynomials in** t **if the initial states** $u(x, 0)$ **grow at infinity like polynomials. However, these solutions are not unique; a requirement of a certain type of** $u(x, 0)$ **at infinity is needed. The one-dimensional classical Phragmén-Lindelöf theorem and some results of Shilov for homogeneous systems are instances of this. It is the purpose here to supply a general (necessary and for some cases sufficient) condition for uniqueness. Preliminary to that a necessary condition is found on** $f(x, t)$ **so that (1) admits solutions that are polynomials in** t.

The one-dimensional classical theorem of Phragmén-Lindelöf can be stated as follows: If u is a soultion of the Cauchy-Riemann equation $\partial u/\partial t = iu/\partial x$ on $R^1 \times R^1$ fulfilling the boundedness condition (2) with $\gamma = \theta < 1$ and

$$(3) \qquad |u(x, 0)| \leqq C(1 + |x|)^\nu, \qquad \nu \geqq 0,$$

then u is a polynomial in (x, t). Therefore, u is identically zero provided condition (3) is replaced by the decay condition on the initial state

$$(4) \qquad u(x, 0) = \theta(|x|^{-d}), \quad d \geqq 0, \quad \text{when} \quad x \to \infty.$$

Shilov [4], [5] or [6] has improved this theorem for a system of N partial differential equation (1) with $f = 0$ under the boundedness conditions (2) and (3). If the eigenvalues of $P(\xi)$ are real for each real vector $\xi \in R^n$, then u has the expression

$$(5) \qquad u(x, t) = \sum_0^r U_k(x)t^k, \qquad r = 2[(n + \nu)/2] + N + 1$$

where the functions U_k are solutions of the systems

$$P(D)U_{k-1} = kU_k, \qquad k = 0, \cdots, r - 1,$$
$$P(D)U_r = 0 \quad \text{on} \quad R^n.$$