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MAXIMAL IDEALS IN THE NEAR RING OF POLYNOMIALS MODULO 2

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A near ring (or semiring) is a structure with addition and composition. Under addition, the structure is a commutative group. Composition is associative and distributive on one side: $(p+q)\circ r = p\circ r + q\circ r$. An example is the set of polynomials with coefficients from the ring of integers [or indeed from any ring]; composition is ordinary composition of polynomials. Another example is the set of endomorphisms of an abelian group.

An ideal in a near ring is, as usual the *kernel of a homomorphism*. (This definition first appeared in G. Birkhoff's 1934 paper, "On the combination of subalgebras," in Proceedings of the Cambridge Philosophical Society.) For $N = \boldsymbol{Z}_2[x, \circ]$, the near ring of polynomials with coefficients from the field $\boldsymbol{Z}_2$ of two elements, the ideal structure is more intricate than it is for $\boldsymbol{Z}_p[x, \circ]$ $(p > 2)$. In this article, all maximal ideals in N are found. Unexpectedly, there are just two of them. There are several other proper ideals. A device due to the referee shows how to construct many of them. Application of his idea is given in the following article.

2. Introduction and summary. The definition of "ideal" shows that, if I is an ideal, then

2.1. I is additively closed:

$$\{t_1 \in I, t_2 \in I\} \Longrightarrow \{t_1 + t_2 \in I\} .$$

2.2. N admits I, in short $I \circ N \subset N$. Explicitly,

$$\{t \in I, n \in N\} \Longrightarrow \{t \circ n \in I\} .$$

2.3. Composition contracts on the right, i.e.,

$$\{t \in I, n_1, n_2 \in N\} \Longrightarrow \{n_1 \circ (n_2 + t) - n_1 \circ n_2 \in I\} .$$

THEOREM 2.4. *Conversely, a subset I is an ideal if it satisfies* 1, 2, 3. (This is a known fact.)

The identity for "$\circ$" is the polynomial x.

Among the results of this article are the following. The set of all polynomials p in N such that $p(0) = p(1)$ is a maximal ideal V,