PACIFIC JOURNAL OF MATHEMATICS
Vol. 53, No. 2, 1974

REPRESENTATIONS OF FINITE RINGS

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In this paper we extend the concept of the Szele representation of finite rings from the case where the coefficient ring is a cyclic ring to the case where it is a Galois ring. We then characterize completely primary and nilpotent finite rings as those rings whose Szele representations satisfy certain conditions.

1. **Preliminaries.** We first note that any finite ring is a direct sum of rings of prime power order. This follows from noticing that when one decomposes the additive group of a finite ring into its pime power components, the component subgroups are, in fact, ideals. So without loss of generality, up to direct sum formation, one needs only to consider rings of prime power order. For the remainder of this paper p will denote an arbitrary, fixed prime and all rings will be of order p^n for some positive integer n. Of the two classes of rings that will be studied in this paper, completely primary finite rings are always of prime power order, so for the completely primary case, there is no loss of generality at all. However, nilpotent finite rings do not need to have prime power order, but we need only classify finite nilpotent rings of prime power order, the general case following from direct sum formation.

If R is finite ring (of order p^n) then the characteristic of R will be p^k for some positive integer k. If $x \in R$ then we define the order of x to be the smallest positive integer e such that $p^e x = 0$. Thus $0 < e \leqq k$.

We now define a very important class of finite rings.

DEFINITION 1.1. Let $f(x) \in Z[x]$, when Z denotes the rational integers, be monic of degree r and irreducible modulo p. Then the ring $Z[x]/(p^k, f(x))$ is called the Galois ring of order p^{kr} and characteristic p^k, and will be denoted by $G_{k,r}$. Basically, then, a Galois ring is an irreducible algebraic extension of degree r of the cyclic ring $Z/(p^k)$, and any two irreducible algebraic extensions of $Z/(p^k)$ of degree r are isomorphic [2, § 3]. Note that $G_{1,r} \cong GF(p^r)$ and $G_{k,1} \cong Z/(p^k)$. This class of rings was introduced independently by Raghavendran [2] and Janusz [1] both of whom called them Galois rings. The importance of Galois rings, at least in our case, is that if R is a completely primary finite ring of characteristic p^k with Jacobson radical J such that $R/J \cong GF(p^r)$ then R contains a unique (up to inner isomorphism) copy of $G_{k,r}$ ([2, Th. 8]). Thus the com-