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ON DEFORMATIONS OF COMPLEX COMPACT MANIFOLDS WITH BOUNDARY

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Let M' be a complex Kählr-Einstein maniford of negative scalar curvature. Let M be a relatively compact submanifold of M' such that $\dim_C M = \dim_C M' = n$ and the boundary bM is a C^∞ submanifold of M' of real dimension $2n-1$. It is further assumed that the following condition holds: There exists a constant $c > 0$ such that for all $\varphi \in C^{0,q}(\bar{M}, \Theta)$, $q = 1, 2$, $((2\Box' - \tilde{\Delta}))\varphi, \varphi) \geqq c(\varphi, \varphi)$ where Θ is the holomorphic tangent bundle of M', $C^{p,q}(\bar{M}, \Theta)$ is the space of all C^∞ Θ-valued (p, q) forms extendible to a neighborhood of $\bar{M}$, $\Box'$ (resp., $\tilde{\Delta}$) is the complex (resp., the real) Laplacian on $C^{p,q}(\bar{M}, \Theta)$ and (,) is the L_2-inner product.

The main result of this paper is that there exists a universal family of deformations of M whose parameter space is, in general, a Banach analytic set. In the case when M is a compact Riemann surface with boundary it is shown that real analytic families of complex structures on M can be described in terms of an open set in $\boldsymbol{R}^m$ where m is the dimension of the reduced Teichmüller space. The proof of this fact is independent of the theory of quasiconformal mappings and Schwarzian derivatives.

The results of the present work are obtained by extending the methods of M. Kuranishi developed for the case of compact manifolds without boundary ([8], [9]). This approach has already been used successfully in the study of complex structures on noncompact manifolds ([6], [7]). The conditions imposed on M' (for a large class of such manifolds we refer to [1]) enable us to use effectively the theory of elliptic boundary value problems (see [5], [10], [11]). Thus we have at our disposal Sobolev k-norm estimates without loss of derivatives. This is crucial for the proof of Proposition 3.6 where, by using the implicit function theorem for Banach spaces, we conclude that every almost complex structure M_φ represented by an element $\varphi \in C^{0,1}(\bar{M}, \Theta)$ with sufficiently small k-norm is isomorphic to an almost complex structure M_ψ with $\bar{\partial}^*\psi = 0$. Moreover, in the proof of our main theorem (Theorem 3.7), the k-norm estimates allow us to use the inverse mapping theorem for Banach spaces and the result of Proposition 3.6 for the actual construction of the universal family.

1. Preliminaries. Let M' be an n-dimensional Riemannian manifold and $M \subset M'$ a subset with nonempty interior such that the boundary bM is a C^∞ submanifold of M' and $\bar{M} = M \cup bM$ is compact.