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STRUCTURAL CONSTANTS II

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The study of finite groups having a Self-Centralizing Sylow subgroup of prime order p is an important part of the theory of finite groups. In this paper these groups are studied under some arithmetical hypotheses. A rational number r, depending on the group and the prime p, is defined and some classification results are obtained by assuming that r is bounded as a function of the prime p.

Let G be a finite group, P a Sylow p-subgroup of G of order p, for an odd prime p.

Fix an element $\pi \in G$ such that $P = \langle \pi \rangle$, and assume $C_G(P) = P$, $q = |N_G(P) : P| = (p-1)/t \neq p-1$, where $C_G(P)$ and $N_G(P)$ denote the centralizer of P in G and the normalizer of P in G, respectively.

Let $\pi = \pi_1, \pi_2, \ldots, \pi_t$ be the representatives of conjugacy classes of elements of order p, where $\pi_i \in P$, $1 \leq i \leq t$. For $1 \leq i, j, k \leq t$, denote by s_{ijk} the number of times a product of a conjugate of π_i, in $N_G(P)$, by a conjugate of π_j, in $N_G(P)$, equals π_k.

Denote by C_{ijk} the number of times a product of a conjugate of π_i, in G, by a conjugate of π_j, in G, equals π_k.

This paper studies the relation between the numbers s_{ijk} and C_{ijk}, $1 \leq i, j, k \leq t$.

Suppose G satisfies the condition

$$(*) \qquad C_{i11} = 0 \quad \text{whenever} \quad s_{i11} = 0, \qquad 1 \leq i \leq t.$$

Define a rational number $r = r(G, p)$ and a rational number $a = a(p, t)$ (p-average of G) as follows:

$$r(G,p) = \max\left\{ \frac{C_{i11}}{s_{i11}} \,\middle|\, \begin{matrix} 1 \leq i \leq t \\ s_{i11} \neq 0 \end{matrix} \right\}$$

$$a(p,t) = \frac{\sum_{i=1}^{t} s_{i11}}{t}$$

This number $r = r(G, p)$ has some interesting properties. For instance, it is true that

(a) $r \equiv 1 (\bmod p)$ as a rational number.