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CHAIN BASED LATTICES

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In recent years several weakenings of Post algebras have been studied. Among these have been P_0-lattices by T. Traczyk, Stone lattice of order n by T. Katrinak and A. Mitschke, and P-algebras by the present authors. Each of these system is an abstraction from certain aspects of Post algebras, and no two of them are comparable. In the present paper, the theory of P_0-lattices will be developed further and two new systems, called P_1-lattices and P_2-lattices are introduced. These systems are referred to as chain based lattices. P_2-lattices form the intersection of all three weakenings mentioned above. While P-algebras and weaker systems such as L-algebras, Heyting algebras, and B-algebras, do not require any distinguished chain of elements other than 0, 1, chain based lattices require such a chain.

Definitions are given in § 1. A P_0-lattice is a bounded distributive lattice A which is generated by its center and a finite subchain containing 0 and 1. Such a subchain is called a chain base for A. The order of a P_0-lattice A is the smallest number of elements in a chain base of A. In § 2, properties of P_0-lattices are given which are used in later sections. If a P_0-lattice A is a Heyting algebra, then it is shown in § 3, that there exists a unique chain base $0 = e_0 < e_1 < \cdots < e_{n-1} = 1$ such that $e_{i+1} \rightarrow e_i = e_i$ for all $i > 0$. A P_0-lattice with such a chain base is called a P_1-lattice. Every P_1-lattice of order n is a Stone lattice of order n. If a P_1-lattice is pseudo-supplemented then it is called a P_2-lattice. It turns out that P_2-lattices of order n are direct products of finitely many Post algebras whose maximum order is n. In § 4, properties of P_2-lattices are studied. In § 5, equational axioms are given for P_2-lattices. P_2-lattices share many of the properties of Post algebras and have application to computer science. Among examples of P_2-lattices are direct products of finitely many p-rings. These further remarks on P_2-lattices are in § 6. In § 7, prime ideals in P_0-lattices are studied. It is shown that the order of a P_0-lattice is one more than the number of elements in a chain of prime ideals of maximum length. A characterization of P_1-lattices by properties of their prime ideals is given. Such a characterization of P_2-lattices is also indicated.

1. DEFINITIONS. We use ϕ for the empty set. Let A be a distributive lattice which is *bounded*, that is, has a largest element 1 and a smallest element 0. The dual of A is denoted by A^d. The