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LINEAR TRANSFORMATIONS ON SYMMETRIC SPACES

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Let U be an n-dimensional vector space over an algebraically closed field F of characteristic zero, and let $\vee^r U$ denote the rth symmetric product space of U. Let T be a linear transformation on $\vee^r U$ which sends nonzero decomposable elements to nonzero decomposable elements. We prove the following:

(i) If $n = r+1$ then T is induced by a nonsingular transformation on T.

(ii) If $2 < n < r+1$ then either T is induced by a nonsingular transformation on U or $T(\vee^r U) = \vee^r W$ for some two dimensional subspace W of U.

The result for $n > r+1$ was recently obtained by L. J. Cummings.

1. Preliminaries. Let U be a finite dimensional vector space over an algebraically closed field F. Let $\bigvee^r U$ denote the rth symmetric product space over U where $r \geqq 2$. Unlese otherwise stated, the characteristic of F is assumed to be zero or greater than r.

A *decomposable subspace* of $\bigvee^r U$ is a subspace consisting of decomposable elements. Let $x_1, \cdots, x_{r-1}$ be $r-1$ nonzero vectors in U. Then the set $\{x_1 \vee \cdots \vee x_{r-1} \vee u : u \in U\}$, denoted by $x_1 \vee \cdots \vee x_{r-1} \vee U$, is a decomposable subspace of $\bigvee^r U$ and is called a *type* 1 *subspace* of $\bigvee^r U$. Let W be a two dimensional subspace of U. It is shown in [2] that $\bigvee^r W$ is decomposable and is called a *type r subspace* of $\bigvee^r U$. If $y_1, \cdots, y_{r-k}$ are vectors in $U - W$ where $1 < k < r$, then the set $\{y_1 \vee \cdots \vee y_{r-k} \vee w_1 \vee \cdots \vee w_k : w_i \in W, i = 1, \cdots, k\}$, denoted by $y_1 \vee \cdots \vee y_{r-k} \vee W \vee \cdots \vee W$, is also decomposable and is called a *type k subspace* of $\bigvee^r U$. In [2] Cummings showed that every maximal decomposable subspce of $\bigvee^r U$ is of type i for some $1 \leqq i \leqq r$.

A linear transformation on $\bigvee^r U$ is called a *decomposable mapping* if it maps nonzero decomposable elements to nonzero decomposable elements. In [3] Cummings proved that if $\dim U > r+1$ then every decomposable mapping T on $\bigvee^r U$ is induced by a nonsingular linear transformation f on U; that is, $T(y_1 \vee \cdots \vee y_r) = f(y_1) \vee \cdots \vee f(y_r)$. In this paper we consider the case when $3 \leqq \dim U \leqq r+1$.

2. The case when $\dim U = r+1$. Two type 1 subspaces M_1 and M_2 of $\bigvee^r U$ are called *adjacent* if

$$M_1 = x_1 \vee \cdots \vee x_{r-2} \vee y_1 \vee U$$
$$M_2 = x_1 \vee \cdots \vee x_{r-2} \vee y_2 \vee U$$