PACIFIC JOURNAL OF MATHEMATICS
Vol. 56, No. 1, 1975

# ON THE EXTENSION OF CONTINUOUS AND ALMOST PERIODIC FUNCTIONS

PAUL MILNES

**The most important results of this paper are two not very closely related theorems concerning the extension of functions. For the first theorem, let $A$ be a subspace of a topological space $B$ and let $X$ and $Y$ be subsets of $C(A)$ and $C(B)$, respectively. The theorem then asserts that, if every member of $X$ extends to a member of $Y$, then every member of the $C^*$-subalgebra of $C(A)$ generated by $X$ extends without increase in norm to a member of the $C^*$-subalgebra of $C(B)$ generated by $Y$. As an application of this theorem, new proofs of some results of J. F. Berglund on the extension of almost periodic functions are given.**

**The statement of the second theorem is: every weakly almost periodic function on an open subgroup $H$ of a locally compact group $G$ extends to a function weakly almost periodic on $G$. (K. deLeeuw and I. Glicksberg have proved this result with the additional assumption that $H$ is normal.)**

**1. Introduction.** A consequence of Pontryagin's duality theorem is as follows:

(A) every continuous character on a closed subgroup $H$ of a locally compact abelian group $G$ admits an extension to a continuous character of $G$ [**5**; (24.12) Corollary, p. 380].

It is not at all clear why (A) should imply

(B) every function in $AP(H)$, the space of almost periodic functions on $H$ (which in this case is just the $C^*$-subalgebra of $C(H)$ generated by the continuous characters on $H$), admits an extension to a function in $AP(G)$.

That (B) does hold is a theorem of Berglund [**1**; Corollary 11]; the result (A) plays an essential role in his proof. That (B) holds may also be deduced from (A) and the following general theorem which is the main result of §2.

THEOREM 1. *Let $A$ be a subspace of a topological space $B$ and let $X$ and $Y$ be subsets of $C(A)$ and $C(B)$ respectively. Then every member of the $C^*$-subalgebra of $C(A)$ generated by $X$ extends without increase in norm to a member of the $C^*$-subalgebra of $C(B)$ generated by $Y$ if*

( $*$ ) *every member of $X$ extends to a member of $Y$*