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TOPOLOGIES COMPATIBLE WITH HOMEOMORPHISM GROUPS

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If $\mathscr{A}$ is a class of open covers of a topological space $(X, \mathscr{T})$, then $(X, \mathscr{T})$ is said to be strongly $\mathscr{A}$-stable provided that for each $\mathscr{C} \in \mathscr{A}$ there is a homeomorphism h mapping X onto X, other than the identity homeomorphism, such that for each $C \in \mathscr{C}$, $h(C) = C$. This paper studies strongly $\mathscr{A}$-stable spaces. Although there are compact connected metric spaces that are not even strongly $\mathscr{A}$-stable with respect to the class $\mathscr{A}$ of all finite open covers, there is an extremely weak homogeneity condition that guarantees that a space $(X, \mathscr{T})$ is strongly $\mathscr{A}$-stable with respect to the class $\mathscr{A}$ of all locally finite open covers. If $H(X)$ is the full homeomorphism group of a space $(X, \mathscr{T})$ that is strongly $\mathscr{A}$-stable with respect to the class $\mathscr{A}$ of all finite open covers, then $H(X)$ is nonabelian and there is a nondiscrete Hausdorff topology $\mathscr{T}$ for $H(X)$ such that $(H(X), \mathscr{T})$ is a topological group.

There has been longstanding interest in topologizing groups of homeomorphisms. For example, in **[16]**, B. L. van der Waerden and D. van Dantzig showed that the group of isometries of a locally compact metric space with only finitely many components is a locally compact topological group under a natural topology; and in [**1**, Theorem 4] R. Arens showed that the full homeomorphism group of a locally compact, locally connected Hausdorff space forms a topological group under the compact open topology. We say that a topology τ on a group $(G, \circ)$ is compatible with $(G, \circ)$ provided that $(G, \circ, \tau)$ is a nondiscrete Hausdorff topological group. It has been known for some time that there exist nondegenerate Hausdorff spaces whose only homeomorphism is the identity. Such spaces are now called rigid spaces, and there are now several constructions of rigid spaces in the literature which differ from the earlier folklore example of a rigid metric continuum **[9]** and **[13]**. These examples show that none of the familiar topological properties on a space X guarantees that the homeomorphism group $H(X)$ admits a compatible topology. On the other hand A. Kertesz and T. Szele have shown that every infinite abelian group admits a compatible topology [**11**, Theorem 1]. The corresponding problem for infinite nonabelian groups is raised in **[11]** and to our knowledge remains unsolved.

In section two we define a sequence of increasingly restrictive topological properties for a space X, each of which is sufficient to