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COEFFICIENT BOUNDS FOR SOME CLASSES OF STARLIKE FUNCTIONS

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Let t be given, $1/4 \leqq t \leqq \infty$, and let $S(t)$ denote the class of normalized starlike univalent functions f in $|z| < 1$ satisfying (i) $|f(z)/z| \geqq t$, $|z| < 1$, if $1/4 \leqq t \leqq 1$, (ii) $|f(z)/z| \leqq t$, $|z| < 1$, if $1 < t \leqq \infty$. If $f(z) = z + \sum_{k=2}^{\infty} a_k z^k \in S(t)$ and n is a fixed positive integer, then the authors obtain sharp coefficient bounds for $|a_n|$ when t is sufficiently large or sufficiently near 1/4. In particular a sharp bound is found for $|a_3|$ when $1/4 \leqq t \leqq 1$ and $5 \leqq t \leqq \infty$. Also a sharp bound for $|a_4|$ is found when $1/4 \leqq t \leqq 1$ or $12.259 \leqq t \leqq \infty$.

1. Introduction. Let S denote the class of starlike univalent functions f in $K = \{z : |z| < 1\}$ with the normalization, $f(0) = 0$, $f'(0) = 1$. Given t, $1/4 \leqq t \leqq \infty$, let $S(t)$ denote the subclass of functions $f \in S$ satisfying

$$|f(z)/z| \geqq t, z \in K, \text{ if } 1/4 \leqq t \leqq 1, \tag{1.1}$$

$$|f(z)/z| \leqq t, z \in K, \text{ if } 1 < t \leqq \infty. \tag{1.2}$$

If $1/4 < t \leqq 1$, we let $F = F(\cdot, t)$ be defined by

$$zF'(z)/F(z) = [1 + 2(2b^2 - 1)z + z^2]^{1/2}/(1 - z), z \in K, \tag{1.3}$$

where $0 \leqq b < 1$ and $t = [(1+b)^{1+b}(1-b)^{1-b}]^{-1}$. The function $F = F(\cdot, t)$ defined by (1.3) is in $S(t)$ for $1/4 < t \leqq 1$, as can be shown by a long but straightforward calculation (see Suffridge **[9]**). For fixed t, $1/4 < t \leqq 1$, this function maps K onto the complex plane minus a set

$$\{w : |w| \geqq t, \quad \pi b \leqq \arg w \leqq 2\pi - \pi b\}.$$

If $1 < t < \infty$, we let $F = F(\cdot, t) \in S(t)$ be defined by

$$\frac{F(z)}{[1 - t^{-1}F(z)]^2} = \frac{z}{(1-z)^2}, z \in K. \tag{1.4}$$

It is well known (see Nehari **[4**, p. 224, ex. 4**]**) that the function F maps K onto a domain whose boundary consists of $\{w : |w| = t\}$, and a slit along the negative real axis from $-t$ to $-\lambda$ where $4\lambda t^2 = (t + \lambda)^2$. If $t = 1/4$ or $t = \infty$, we let