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COMPLETION AND SEMICOMPLETION OF MOORE SPACES

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Two properties are given which characterize those metacompact Moore spaces that are completable and two which characterize those which are semicompletable. The relationship among these properties in nonmetacompact spaces is investigated. Two new characterizations of Moore-closed spaces are given as is a characterization of developability. Some unification of technique is achieved among the various ways of completing or semicompleting a Moore space.

In 1962, O. H. Alzoobaee stated two properties A and B of developments and showed that every Moore space having a development with property (A) B is (semicompletable) completable and every (semicomplete) complete Moore space has a development with property (A) B. In the present work, it is shown that every *metacompact* (semicompletable) completable Moore space has a development with property (A) B. Related properties A' and B' are investigated. It is shown that a Moore space is Moore-closed iff *every* strong development has property A and iff every strong development has property B. Finally, a developable topological space is shown to have a sequence $\langle G_n \rangle$ of open covers such that for each point p and open set U containing p, there is an n such that only one member of G_n contains p and it is a subset of U.

Complete Moore spaces were introduced in [7] and it was shown in [12] and [17] that a metrizable space is metrically topologically complete iff it is a complete Moore space. Semicompleteness (sometimes called Rudin completeness), a weaker property, was introduced in [14] and shown to be equivalent to completeness in metrizable spaces. Unlike the theories of metrizable or uniformizable spaces, there exist Moore spaces that cannot be embedded in any semicomplete Moore space [14], [9] and there exist semicomplete Moore spaces that cannot be completed [14]. The question naturally arises then of characterizing (semi-) completable Moore spaces. Whipple [15] has characterized completable Moore spaces in terms of Cauchy sequences and Creede [3] has what might be called an external characterization in terms of the Wallman compactification. An external characterization of semicompletability is given by J. N. Reed in [11]. Stronger completeness properties are investigated in [4] and [5]. The properties of Alzoobaee mentioned above were introduced in [1] and reported