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ON BOUNDED SOLUTIONS OF A STRONGLY NONLINEAR ELLIPTIC EQUATION

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I. Introduction. Consider the Dirichlet problem for a bounded domain $G \subset R^n (n \geqq 2)$ having smooth boundary ∂G:

$$(1) \qquad \begin{aligned} &\mathscr{A}u + p(u) = -D_i f_i + f \\ &z|_{\partial G} = 0 , \end{aligned}$$

where $\mathscr{A}$ is a second order differential operator of Leray-Lions type mapping a real Sobolev space $W_0^{1,q}(G)(1 < q < \infty)$ into its dual; $f, f_i (i = 1, \cdots, n)$ are given functions. We have used the notation D_i for the derivative in the distribution sense $\partial/\partial x_i$ and the convention that if an index is repeated then summation over that index from 1 to n is implied. We shall assume that the real function $p(t)$ is continuous and satisfies the condition

$$(2) \qquad p(t)t \geqq 0 \qquad \forall t \in R ,$$

but otherwise $(p)t$ is not subject to any growth condition.

In this paper we discuss the existence of a solution of equation (1) in $W_0^{1,q}(G) \cap L^\infty(G)$.

Many papers appearing recently have studied equations and inequations involving strongly nonlinear elliptic operators of the type (1). For equations we mention among others [1], [2], [7]; in [1] and [2] the existence of a solution in $W_0^{m,q}(G)$ when the operator $\mathscr{A}$ has arbitrary order $2m$ is established under the additional hypothesis:

Given $\varepsilon > 0$, there exists $K_\varepsilon > 0$ such that

$$(3) \qquad p(t)s \leqq \varepsilon p(s)s + K_\varepsilon[1 + p(t)t] \qquad \forall t, s \in R$$

[3], [9] among others deal with strongly nonlinear inequations in $W^{m,q}(G)$.

For an operator $\mathscr{A}$ of second order, [4] proves the existence of a solution in $W_0^{1,q}(G)$ under the sole condition (2).

Finally let us mention that the existence of bounded solution of other strongly nonlinear equations and inequations has been discussed in [8]. However it seems to us that the technique of this paper is different from ours; it consists of multiplying the equation with a nonlinear expression of u; it also seems that our method when applied to some concrete cases yields different results in the sense that we only require the functions in the right hand side of (1) to be in $L^r(G)$ for some $r > 1$ and not in $L^\infty(G)$ as in [8].