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CONCERNING σ-HOMOMORPHISMS OF RIESZ SPACES

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If L is a Riesz space (lattice ordered vector space), a Riesz homomorphism of L is an order preserving linear map which preserves the finite operations "$\vee$" and "$\wedge$". It was shown in our previous paper ["Homomorphisms of Riesz spaces," Pacific J. Math.] that there is a large class α of spaces such that if L belongs to α and φ is a Riesz homomorphism from L onto an Archimedean Riesz space, then φ preserves the order limit of sequences. In this paper the list of members of α is extended. It is further shown that there is a large class β of spaces with the property that if L belongs to α and φ is a Riesz homomorphism of L into an Archimedean Riesz space then φ preserves the order limit of sequences.

This paper is a continuation and extension of Tucker [4]. The notation and terminology of Tucker [4] will be used.

LEMMA 1. *Suppose L is a Riesz space with the principal projection property, K is an Archimedean Riesz space, and φ is a Riesz homomorphism of L into K with the property that if $\{b_1, b_2, b_3, \cdots\}$ is a countable orthogonal subset of L^+ such that $b = \bigvee b_i$, then $\varphi(b - \sum_{i=1}^{j} b_i) \rightarrow \theta$, then φ preserves the order limits of sequences.*

Proof. Suppose $f_1, f_2, f_3, \cdots$ is a sequence of points of L such that $f_1 \geqq f_2 \geqq f_3 \geqq \cdots \geqq \theta$ and $\bigwedge f_i = \theta$. Suppose, further, that n is a positive integer. For each i let $g_i = f_i - (1/2^n)f_1$, $h_i = \bigvee_p (pg_i^+ \wedge g_1)$, and $b_i = h_i - h_{i+1}$. Consider b_i and b_j where $j > i$. Now $b_j \leqq h_{i+1}$ and $b_i \leqq g_1 - h_{i+1}$, so that $\theta = h_{i+1} \wedge (g_1 - h_{i+1}) \geqq b_j \wedge b_i$. Thus $\{b_1, b_2, b_3, \cdots\}$ is a countable orthogonal set.

Since $g_1 \geqq b_i$ for each i, g_1 is an upper bound of $\{b_1, b_2, b_3, \cdots\}$. Suppose α is a point such that $g_1 - \alpha \geqq b_i$ for each i and $\alpha \geqq \theta$. Let i be a positive integer and let β be the projection of α on b_i. Then $g_1 - \beta \geqq b_i$. Now $b_i + \sum_{j=1}^{i-1} b_j + h_{i+1} = h_1 = g_1$ and $b_i \wedge (\sum_{j=1}^{i-1} b_j + h_{i+1}) = \theta$ so that $\beta \wedge (g_1 - b_i) = \theta$. Since $g_1 - \beta \geqq b_i$, $g_1 - b_i \geqq \beta$ which implies $(g_1 - b_i) \wedge \beta = \beta$, so that $\beta = \theta$. Thus for each i, $\alpha \wedge b_i = \theta$. Now $g_1 \geqq b_i$ so that $g_1 \geqq \bigvee_{j=1}^{i} b_j \vee \alpha = \sum_{j=1}^{i} b_j + \alpha = h_1 - h_{i+1} + \alpha = g_1 - h_{i+1} + \alpha$ which implies $h_{i+1} \geqq \alpha$.

Now $g_i^- \wedge g_i^+ = \theta$ which implies $g_i^- \varepsilon (g_i^+)^d$. As h_i is the projection of g_1 on g_i^+, $h_i \wedge g_i^- = \theta$.

Without loss of generality we may assume that $(1/2^n)f_1 \geqq \alpha$. Then