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MAXIMAL CONNECTED HAUSDORFF SPACES

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A nowhere neighborhood nested space is one in which no point has a local base which is linearly ordered by set inclusion. An *MI* space is one in which every dense subset is open. *In this paper we show that every Hausdorff topology* without isolated points has a nowhere neighborhood nested refinement. We show that every maximal connected Hausdorff topology is *MI* and nowhere neighborhood nested, and that every connected, but not maximal connected, Hausdorff topology has a connected, but not maximal connected, nowhere neighborhood nested refinement. Every connected Hausdorff topology has a connected, *MI*, nowhere neighborhood nested refinement.

In [4] the author raised the question of the existence of nontrivial maximal connected Hausdorff spaces. The question remains open.

A topology $\mathscr{T}'$ on a set X is said to be finer than, or to be a refinement of, a topology $\mathscr{T}$ on X if $\mathscr{T} \subset \mathscr{T}'$. It is said to be strictly finer than $\mathscr{T}$, if, in addition, we have $\mathscr{T} \neq \mathscr{T}'$ We say that $(X, \mathscr{T})$ (and by abuse of language, $\mathscr{T}$) is maximal connected, if $(X, \mathscr{T})$ is connected and whenever $\mathscr{T}'$ is strictly finer than $\mathscr{T}$, $(X, \mathscr{T}')$ is not connected. An MI space (see [2]) is one is which every dense subset is open.

The following result is in the authors thesis [5].

THEOREM 1. *Every maximal connected space is an MI space.*

An irresolvable space is one which does not have a dense subset whose complement is also dense. Anderson [1] has shown that every connected Hausdorff space has a connected irresolvable refinement. If in his proof of his Theorem 1, in the fourth paragraph, we simply choose D to be an R^*-dense set which is not R^* open, we will have proved

THEOREM 2. *Let τ be an infinite cardinal number. Let R be a connected topology for X with $\Delta(R) \geqq \tau$, where $\Delta(R)$ denotes the dispersion character, or minimum cardinality of an open set of R. Then there exists a connected MI refinement R^* of R with $\Delta(R^*) \geqq \tau$.*

DEFINITION 1. Let $(X, \mathscr{T})$ be a topological space, $x \in X$. If there is a "local" base at x which is linearly ordered under set