PACIFIC JOURNAL OF MATHEMATICS
Vol. 57, No. 2, 1975

AN ASYMPTOTIC ANALYSIS OF AN ODD ORDER LINEAR DIFFERENTIAL EQUATION

DAVID LOWELL LOVELADY

Let q be a continuous function from $[0, \infty)$ to $(0, \infty)$, and let n be a positive integer. With respect to the equation $u^{(2n+1)} + qu = 0$, we study the relationship between the existence of oscillatory solutions and the asymptotic behavior of nonoscillatory solutions.

There is no additional hypothesis on q which will ensure that every solution of

$$u^{(2n+1)} + qu = 0 \tag{1}$$

is oscillatory. In particular, it follows from a result of P. Hartman and A. Wintner [5] that there is a solution u of (1) such that

$$(-1)^k u^{(k)}(t) > 0 \tag{2}$$

whenever $t \geqq 0$ and $0 \leqq k \leqq 2n$. We shall call a solution of u of (1) *strongly decreasing* if and only if there is $c \geqq 0$ such that (2) is true whenever $t \geqq c$ and $0 \leqq k \leqq 2n$. Since we know that (1) has a strongly decreasing solution, the best result one can hope for in an oscillation theorem is that every eventually positive solution of (1) is strongly decreasing. G. V. Anan'eva and V. I. Balaganskii [1] (see also C. A. Swanson [7, p. 175]) have shown that if

$$\int_0^\infty t^{2n-1} q(t)dt = \infty\ , \tag{3}$$

then every eventually positive solution of (1) is strongly decreasing. Our first result extends this.

THEOREM 1. *If* (3) *fails and the second order equation*

$$w''(t) + \frac{1}{(2n-2)!}\left(\int_t^\infty (s-t)^{2n-2} q(s)ds\right)w(t) = 0 \tag{4}$$

is oscillatory, then every eventually positive solution of (1) *is strongly decreasing.*

Although the conclusion of Theorem 1 limits the asymptotic behavior of nonoscillatory solutions of (1) (if u is nonoscillatory then either u or $-u$ is eventually positive), it does not in fact ensure the existence of oscillatory solutions.