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BEHAVIOR OF Φ-BOUNDED HARMONIC FUNCTIONS AT THE WIENER BOUDARY

YOUNG K. KWON

For a strongly convex $\Phi(t)$, denote by $H\Phi$ the class of Φ-bounded harmonic functions, and by $C\Phi$ the class of continuous functions f on the Wiener harmonic boundary such that the composite $\Phi(|f|)$ is integrable with respect to a harmonic measure. Theorem: $u \in H\Phi$ if and only if u is a solution of the Dirichlet problem with boundary values $f \in C\Phi$ on the Wiener harmonic boundary.

1. For a strongly convex $\Phi(t)$ Naim [4] developed an integral representation of Φ-bounded harmonic functions in terms of the Martin minimal boundary and fine topology, and for $\Phi(t) = t^p$ $(p > 1)$ Schiff [7] extended the results in the framework of the Wiener compactification. In view of the fact that the Wiener compactification is "smaller" than the Martin, the latter sharpens the former.

The purpose of the present paper is to show that Naim's theory of Φ-bounded harmonic functions does carry over in its full generality to the Wiener compactification setting.

2. The set-up is, as in Naim [4], a locally compact, noncompact, connected and locally connected Hausdorff space Ω with a Brelot harmonic sheaf H such that 1 is superharmonic (cf. Brelot [1]). An increasing nonnegative function $\Phi(t)$ on $[0, \infty)$ is said to be strongly convex if $\Phi(t)$ is convex and $\lim_{t\to\infty} t^{-1}\Phi(t) = \infty$. Following Parreau [5], a harmonic function u is said to be Φ-bounded if the function $\Phi(|u|)$ has a harmonic majorant, and u is said to be quasibounded if $u = u_1 - u_2$ where u_1, u_2 are limits of nondecreasing sequences of nonnegative bounded harmonic functions.

Throughout this paper we base our arguments on the Wiener compactification (cf. Sario and Nakai [6], and Loeb and Walsh [3]). Let Δ be the Wiener harmonic boundary, $P(x, t)$ the harmonic kernel, and μ the harmonic measure (centered at $x_0 \in \Omega$, say). It is now classic that u is quasibounded if and only if $u(x) = \int_\Delta P(x, t)f(t)d\mu(t)$ for some μ-integrable function f on Δ. In this case u has a continuous extension to Δ and $u = f$ μ-a.e.

3. Denote by $H\Phi(\Omega)$ the class of Φ-bounded harmonic functions on Ω, and by $C\Phi(\Delta)$ the class of extended real-valued continuous functions f on Δ such that $\Phi(|f|)$ is μ-integrable.