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IMAGES AND PRE-IMAGES OF LOCALIZATION MAPS

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This paper treats the induced function $\mathscr{L}_*: [X, Y] \to [X, Y_P]$ when X and Y are spaces having "just enough" structure and $\mathscr{L}: Y \to Y_P$ is a localization map. Special attention is paid to the images of finite subsets of $[X, Y]$ and the preimages of finite subsets of $[X, Y_P]$. While our results are not restricted to finite $[X, Y]$ we do, as an immediate corollary of our main theorem, establish necessary and sufficient conditions for the finiteness of $[X, Y_P]$.

1. Introduction. If Y is a homotopy-associative H-space[1] it is relatively easy to study properties of $\mathscr{L}_*: [X, Y] \to [X, Y_P]$ where P is a set of prime integers and $\mathscr{L}: Y \to Y_P$ is a localization map [1], [7]. The most obvious technique to use is to note that $[X, Y]$ and $[X, Y_P]$ are nilpotent groups, $\mathscr{L}_*$ is a homomorphism and $[X, Y_P]$ is isomorphic to $[X, Y]_P$, the localization of the group $[X, Y]$. For example if Y is a homotopy associative H-space of type $(n_1, \cdots, n_r)$ and X is finite CW, it is immediate that $[X, Y]$ is finite if, and only if $\bigoplus_{i=1}^r H^{n_i}(X; Q) = 0$. Even without finiteness restrictions there are numerous results which can be based on the structure of finitely generated nilpotent groups [2].

At the other exteme, there is in general little that one can say about the system $\mathscr{L}_*: [X, Y] \to [X, Y_P]$ if there are no restrictions on Y. A notable exception to this statement are the results of Hilton, Mislin and Roitberg [3].

In the present paper, while some of our results are general and overlap [3], we are primarily concerned with spaces Y which have "just enough structure." This class of spaces is essentially the p-universal spaces of [6], and in particular contains all rational H-spaces and co-H-spaces (that is, spaces whose localization at the empty set is an H-space or co-H-space). While our results are not restricted to finite $[X, Y]$ we do, as an immediate corollary of our main theorem, prove:

Let Y be a finitely generated rational H-space or co-H-space, let X be a finite CW-space and let $\mathscr{L}: Y_P \to Y_S$ be a localization map ($P \supset S$ are subsets, not necessarily proper, of the set of prime integers). Then $[X, Y_P]$ is finite if, and only if $[X, Y_S]$ is finite. Furthermore, $\mathscr{L}_*: [X; Y_P] \to [X; Y_S]$ is epic.

[1] Unless otherwise noted all spaces are assumed to be of finite type and simple, or the P-localizations of such spaces.