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ABSOLUTELY DIVERGENT SERIES AND ISOMORPHISM OF SUBSPACES

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We consider the relation between the following two statements for E and F a pair of normed spaces.

(SI) For each absolutely divergent series $\Sigma_n x_n$ in E there is a continuous linear mapping T from E into F such that $\Sigma_n Tx_n$ diverges absolutely.

(LI) The finite dimensional subspaces of E are uniformly isomorphic to subspaces of F under isomorphisms which extend to all of E without increase of norm.

Our main result is that (SI) implies (LI) when F is isometric to $F \times F$ with a certain type of norm. We also observe that if a normed space E is not isomorphic to a subspace of an $L_p(\mu)$ space, then for each r with $1 \leqq r < \infty$ there is a series $\Sigma_n x_n$ in E such that $\Sigma_n \|Tx_n\|^r < \infty$ for each continuous linear mapping T from E into l_p but $\Sigma_n \|x_n\|^r = \infty$.

It is not hard to show that (LI) $\Rightarrow$ (SI) (Proposition 4.1). The main thrust of our work is to prove that (SI) $\Rightarrow$ (LI) in some important cases when F has infinite dimension. (Theorems 4.2 and 4.6). Our most important result is Theorem 4.6 which roughly maintains that (SI) $\Rightarrow$ (LI) if F is uniformly isometric to $F \times F$ in a way which we shall later clarify (Definition 4.5). The condition we need on F is satisfied for most familiar Banach spaces (e.g. l_p, $L_p[0, 1]$, $(1 \leqq p \leqq \infty)$, $C[0, 1]$).

Sections §2 and §3 are devoted to a basic study of properties (SI) and (LI) respectively. In §5 we relate our work here with that of other authors and state some problems.

2. Series immersion.

DEFINITION 2.1. A normed space E is said to be *series immersed* in a normed *space* F if the following statement holds:

(SI) For each absolutely divergent series $\Sigma_n x_n (\Sigma_n \|x_n\| = \infty)$ in E there is a continuous linear mapping T from E into F such that $\Sigma_n Tx_n$ diverges absolutely.

If E is series immersed in F then each subspace of E is also. An easy perturbation argument shows that E the completion of E is also series immersed in F.