

EXTENSION OF CONTINUOUS FUNCTIONS ON TOPOLOGICAL SEMIGROUPS

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Examples show that functions of various kinds on subsemigroups of topological semigroups do not always extend to functions of the same kind on the containing semigroup. We show here that, if S is a dense subsemigroup with identity of a topological group G , then there is a fairly large subspace of $C(S)$ whose members always extend at least to members of $C(G)$. As important applications of this theorem, we prove in this setting that the weakly almost periodic functions on S extend to functions weakly almost periodic on G and, in a somewhat more restricted setting, that the weakly almost periodic functions on S are uniformly continuous. These results broaden the scope of answers we gave recently to some questions posed by R. Burckel. We also prove variants of some recent results of A. T. Lau and of S. J. Wiley, results concerning the extension of functions and the existence of invariant means on dense subsemigroups of topological groups.

1. Introduction. Suppose S is a subsemigroup of a topological semigroup T . It follows directly from the definitions that the restriction to S of a function continuous (left uniformly continuous) [weakly almost periodic] {almost periodic} on T is continuous (left uniformly continuous) [weakly almost periodic] {almost periodic} on S . However, examples show that the converse is not true in general: a function of a specified kind on S does not always extend to a function of the same kind on T . In this paper we show that, if S is a dense subsemigroup with identity of a topological group G , then there is a subspace of $C(S)$ whose members always extend at least to members of $C(G)$. This subspace is fairly large — it always contains the left uniformly continuous functions — and was first introduced by T. Mitchell in his study [14] of the connection between fixed point properties that a topological semigroup S might conceivably possess and the existence of different kinds of invariant means on subspaces of $C(S)$.

As applications of the extension theorem, which is itself a variant of a theorem of A. T. Lau [11], we prove variants of some recent results of S. J. Wiley [17]. More important applications are proofs that, if S is a dense subsemigroup with identity of a topological group G , then every weakly almost periodic function on S extends to a function weakly