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STIELTJES DIFFERENTIAL-BOUNDARY OPERATORS III, MULTIVALUED OPERATORS-LINEAR RELATIONS

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This article deals with a multivalued differential-boundary operator on a nondense domain regarding it as a linear relation. The adjoint relation is derived. It is shown that these dual relations have the same form as exhibited in earlier papers where the operators involved were uniquely defined on dense domains. Self-adjoint relations are considered on the Hilbert space $\mathscr{L}_n^2[0, 1]$. The connection with self-adjoint operators defined on subspaces of $\mathscr{L}_n^2[0, 1]$ is made.

I. Introduction. This article is a continuation of [8] and [9]. The notation is the same. We review it briefly. X is the Banach space $\mathscr{L}_n^p[0, 1]$, $1 \leqq p < \infty$, consisting of all n-dimensional vectors

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

under the norm

$$\| y \| = \left[\int_0^1 \left[\sum_{i=1}^n | y_i |^2\right]^{p/2} dt\right]^{1/p} .$$

X^* is the dual space $\mathscr{L}_n^q[0, 1]$, $1/p + 1/q = 1$.

A and B are $m \times n$ matrices, $m \leqq 2n$, satisfying rank $(A\colon B) = m$. C and D are $(2n - m) \times n$ matrices such that $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ is nonsingular. $\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1}$ is given by $\begin{pmatrix} -\tilde{A}^* & -\tilde{C}^* \\ \tilde{B}^* & \tilde{D}^* \end{pmatrix}$, where $\tilde{A}$ and $\tilde{B}$ are $m \times n$ matrices satisfying rank $(\tilde{A}\colon \tilde{B}) = m$, and $\tilde{C}$ and $\tilde{D}$ are $(2n - m \times n)$ matrices. Hence the large matrices above may be multiplied together in the usual component-like manner.

K is a regular $m \times n$ matrix valued function of bounded variation satisfying $dK(0) = 0$, $dK(1) = 0$. K_1 is a regular $r \times n$ matrix valued function of bounded variation satisfying $dK_1(0) = 0$, $dK_1(1) = 0$.

H is a regular $n \times (2m - m)$ matrix valued function of bounded variation satisfying $dH(0) = 0$, $dH(1) = 0$. H_1 is a regular $n \times s$ matrix valued function of bounded variation satisfying $dH_1(0) = 0$, $dH_1(1) = 0$. P is a continuous $n \times n$ matrix.

Now let $\mathscr{D}$ denote those elements $y \in X$ satisfying

1. For each y there is an $s \times 1$ matrix valued constant ψ such that