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THE BRAUER GROUP OF POLYNOMIAL RINGS

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Let R be a commutative ring and S a commutative R-algebra. The induced homomorphism $B(R) \to B(S)$ of Brauer groups is studied for the following choices of S. First, $S = R/I$ where I is an ideal in the radical of R. Second, $S = R[x]$ the ring of polynomials in one variable over R. Third, $S = K$ the quotient field of R when R is a domain.

In [3] M. Auslander and O. Goldman introduced the Brauer group $B(R)$ of a commutative ring R. If S is a commutative R-algebra there is a homomorphism $B(R) \to B(S)$ induced by the homomorphism from R to S. Some of the choices for S considered in [3] are $S = R/I$ for an ideal I of R, or $S = K$ the quotient field of R when R is a domain, or $S = R[x]$ the ring of polynomials in one variable over R.

We observe here relationships between the homomorphisms of Brauer groups induced from these choices for S. We show that if I is an ideal in the radical of R and R is complete in its I-adic topology then $B(R) \cong B(R/I)$. This answers a question raised in [11]. If I is a nil ideal in R then $B(R) \cong B(R/I)$. If $R[[x]]$ is the ring of formal power series over R then $B(R[[x]]) \cong B(R)$. If we assume R is a domain with quotient field K an algebraic number field and $t_1, \cdots, t_n$ are indeterminates the homomorphism $B(R[t_1, \cdots, t_n]) \to B(K(t_1, \cdots, t_n))$ is a monomorphism where $K(t_1, \cdots, t_n)$ is the function field in n-variables over K. Let $B'(R[x])$ be the kernel of the natural homomorphism $B(R[x]) \to B(R)$ where x is an indeterminate. If R is a domain there is a procedure given in [13] for calculating $B'(R[x])$ in terms of $B'(\bar{R}[x])$ where $\bar{R}$ is the integral closure of R. In [3] it is shown that $B'(R[x]) = 0$ if R is a regular domain of characteristic $= 0$. We fill in the gap between these two results in the Noetherian case.

If R is an integrally closed Noetherian domain, let Ref (R) denote the isomorphism classes of finitely generated reflexive R-modules M with $\mathrm{End}_R(M)$ projective over R and let Pro (R) be the projective elements in Ref (R). Under the multiplication $|M| \cdot |N| = |(M \otimes N)^{**}|$ Ref (R) is a monoid, Pro (R) is a submonoid and Ref (R)/Pro (R) is a group (see [6]). There is a split exact sequence.

$0 \to \mathrm{Ref}'(R[x]) \to \mathrm{Ref}(R[x])/\mathrm{Pro}(R[x]) \to \mathrm{Ref}(R)/\mathrm{Pro}(R) \to 0$ where $\mathrm{Ref}'(R[x]) = \mathrm{Ref}(R[x])/(\mathrm{Pro}(R[x]) + \mathrm{Ref}(R))$. Utilizing results in [1] we show that the sequence.

$0 \to \mathrm{Ref}'(R[x]) \to B'(R[x]) \to B'(K[x])$ is exact. If R is any von