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PLESSNER'S THEOREM FOR RIESZ CONJUGATES

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Plessner's theorem states that if a trigonometric series converges everywhere in a set E of positive measure, then its conjugate series converges almost everywhere in E. Recently, Ash and Gluck have shown that this theorem is false in two dimensions by exhibiting a Fourier series of an L^1 function which converges almost everywhere, but each of its conjugates is divergent almost everywhere. We show that if instead of the usual conjugates in two dimensions, one uses Riesz conjugates, then Plessner's theorem remains true provided the conjugates are required only to be restrictedly convergent almost everywhere in E. The techniques used to obtain this result are similar to those used in the one-dimensional case and involve the notions of stable convergence, nontangential convergence, the theory of Riesz conjugates as developed by E. M. Stein and G. Weiss, and a Tauberian theorem for Abel summability.

1. Introduction. In [1], J. M. Ash and L. Gluck presented some results for Fourier series in several variables. They proved in dimension 2 that each of the conjugate series of a Fourier series of a function in $L^p (p > 1)$ converges almost everywhere in the set where the Fourier series converges. In the case $p = 1$, however, they exhibited a function whose Fourier series converges almost everywhere such that each of its conjugates is also a Fourier series of an L^1 function, but is square divergent almost everywhere. Furthermore, in dimension 3 or greater, they found a continuous function whose Fourier series converges almost everywhere such that each of its conjugates is also a Fourier series of a continuous function, but is restrictedly divergent almost everywhere.

On a philosophical level, this distressing state of affairs can be explained by the fact that the "singularity" of each conjugate transformation they use, thought of as a "singular integral operator", has changed from a point to a pair of lines as the dimension of the space was increased from 1 to 2. This can be altered by using instead of the ordinary conjugate series, the Riesz conjugates. This is done also to take advantage of the theory of conjugate transformations developed by Stein and Weiss in [3] or [4] and [5]. By doing this, we are able to retain Plessner's theorem in its original form except that the conjugates will be