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RINGS WITH QUASI-PROJECTIVE LEFT IDEALS

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A ring R is a left qp-ring if each of its left ideals is quasi-projective as a left R-module in the sense of Wu and Jans. The following results giving the structure of left qp-rings are obtained. Throughout R is a perfect ring with radical N: (1) Let R be local. Then R is a left qp-ring iff $N^2 = (0)$ or R is a principal left ideal ring with dcc on left ideals, (2) If R is a left qp-ring and T is the sum of all those indecomposable left ideals of R which are not projective, then T is an ideal of R and $N = T \oplus L$, L is a left ideal of R such that every left subideal of L is projective, R/T is hereditary, and R is heredity iff $T = (0)$. (3) If R is left qp-ring then $R = \begin{pmatrix} S & M \\ 0 & T \end{pmatrix}$, where S is hereditary, T is a direct sum of finitely many local qp-rings and M is a (S, T)-bimodule. (4) A perfect left qp-ring is semi-primary. (5) Let R be an indecomposable ring such that it admits a faithful projective injective left module. Then R is a left qp-ring iff R is a local principal left ideal ring or R is a left-hereditary ring with dcc on left ideals. (6) Let R be an indecomposable QF-ring. Then R is a left qp-ring if each homomorphic image of R is a q-ring (each one-sided ideal is quasi-injective). (7) If a left ideal A of left qp-ring R is not projective then the projective dimension of A is infinite, thus $lgl.\ \dim R = 0, 1$, or ∞. An example of a left artinian left qp-ring which is not right qp-ring is also given.

Clearly all left hereditary rings are left qp-rings. However, the class of commutative principal ideal artinian rings which are not direct sum of fields distinguishes qp-rings from hereditary rings. Commutative pre-self-injective rings studied by Klatt and Levy **[8]** and by Levy **[11]** form a class dual to the class of commutative qp-rings. Dual to the noncommutative qp-rings are rings for which every cyclic module is quasi-injective investigated by Ahsan **[1]** and by Koehler **[9]**. In this paper we study perfect left qp-rings.

2. A ring R is said to be right (left) perfect if it satisfies dcc on principal left (right) ideals and R is called perfect if it is both right and left perfect **[3]**. An artinian principal ideal ring is called uniserial.

A ring R with Jacobson radical N is called local if R/N is a division ring. We assume that all nonzero rings have nonzero identity elements