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COMPACT SUBSETS OF A TYCHONOFF SET

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The paper establishes a relation between the partial exponential law and the compactness of certain subsets of Tychonoff sets of multifunctions, and deduces consequences bearing on the Ascoli theorems established by Weston and Lin-Rose.

1. Introduction. The "Tychonoff set" is an abstraction of a class of sets arising in extensions of the classical Tychonoff theorem to multifunction context ([2], [5]). Extending the definition of the partial exponential law to multifunctions, we show that, when it is satisfied for a topology τ, certain subsets of a Tychonoff set are τ-compact. This approach — which is a non-trivial modification of the method introduced into function Ascoli theory by Noble [7] — will yield, in particular, sufficient conditions for compactness relative to the compact open topology.

In [6] Lin and Rose introduced a multifunction extension of the Kelley-Morse notion of even continuity, and proved a multifunction Ascoli theorem of the Weston type, without, however, showing that it contains the prototype [11, p. 20]. We deduce from our criterion a generalization of the Lin-Rose theorem. We show that this generalization contains the Weston Ascoli theorem and yields corollaries equivalent to the Tychonoff theorems for point-compact and point-closed multifunctions established in [2].

2. Multifunctions. We review the established definitions for multifunctions ([1], [9], [10]): Let X, Y be nonempty sets. A *multifunction* is a point to set correspondence $f: X \to Y$ such that, for all $x \in X, fx$ is a nonempty subset of Y. For $A \subseteq X, B \subseteq Y$ it is customary to write $f(A) = \bigcup_{x \in A} fx, f^-(B) = \{x : x \in X \text{ and } fx \cap B \neq \phi\}$ and $f^+(B) = \{x : x \in X \text{ and } fx \subseteq B\}$. If Y is a topological space, a multifunction $f: X \to Y$ is *point-compact* (*point-closed*) if fx is compact (closed) for all $x \in X$. If X, Y are topological spaces, a multifunction $f: X \to Y$ is *continuous* if $f^-(U), f^+(U)$ are open in X whenever U is open in Y. Henceforth the set of all continuous multifunctions (continuous functions) on a topological space X to a topological space Y will be denoted $\mathscr{C}(X, Y)(C(X, Y))$.

Let $\{Y_x\}_{x \in X}$ be a family of nonempty sets. The *m-product* $P\{Y_x : x \in X\}$ of the Y_x is the set of all multifunctions $f: X \to \bigcup_{x \in X} Y_x$