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ON THE GROUPS OF UNITS IN SEMIGROUPS OF PROBABILITY MEASURES

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We generalize Pym's decomposition $w = \mu_E * w_H * \mu_F$ of idempotent probability measures to the decomposition $\mu_E * \mathcal{H}(w_H) * \mu_F$ of the maximal groups of units in semigroup of probability measures on a compact semitopological semigroup. We also prove that $\mathcal{H}(w) \cong \mathcal{H}(w_H) \cong N(H)/H$ algebraically and topologically. With these characterizations, we verify Rosenblatt's necessary and sufficient condition for the convergence of a convolution sequence $(\nu^n)_{n \geqq 1}$ of a probability measure ν on a compact topological semigroup.

1. Introduction. Let S denote a compact semitopological semigroup (i.e., the multiplication is separately continuous) and $(C(S), \| \ \|)$ the Banach space of all bounded real-valued continuous functions on S. Then $M^b(S)$ which is defined as the norm dual of $C(S)$ is a Banach algebra under $\|\mu\| = \sup\{|\mu(f)| : \|f\| \leqq 1\}$ and the convolution $*$ which is defined via $\mu * \nu(f) = \int f(xy)\mu(dx)\nu(dy)$ on $C(S)$. Let $P(S)$ be the totality of probability measures on S, which consists of all positive measures with norm 1 in $M^b(S)$. Then $P(S)$ is a compact semitopological semigroup under $*$ and the weak* topology which is the topology of pointwise convergence on $C(S)$ **[4]**. If S is topological (i.e., the multiplication is jointly continuous), then $P(S)$ is topological (Prop. 4, **[9]**).

It is known that every compact semitopological semigroup has a minimal ideal which is not necessarily closed except in the case S is topological [7]. We thus introduce the following definition:

> A compact semitopological semigroup is called topologically simple if its minimal ideal is dense in it.

For a subsemigroup T of S, we use $E(S)$ and $M(T)$ to denote the totality of idempotents and the minimal ideal in S respectively. For a subsemigroup A *of* $P(S)$, we write $D(A) = \cup\{\operatorname{supp} \mu : \mu \in A\}$ and $\operatorname{supp} A = \overline{D(A)}$, where $\operatorname{supp} \mu$ denotes the support of μ.

In the remainder, S will always denote a compact semitopological semigroup except mentioned especially.