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CONVOLUTION MULTIPLIERS AND DISTRIBUTIONS

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In this paper, in a purely algebraic way, Schwartz distributions in several variables are generalized in accordance with their homomorphism interpretation proposed by R. A. Struble.

0. Introduction. R. A. Struble in **[10]** has shown that Schwartz distributions can be characterized simply as mappings, from the space $\mathscr{D}$ of test functions into the space $\mathscr{E}$ of smooth functions, which commute with ordinary convolution. This new view of distributions has turned out to be very useful **[11, 12]** and motivated us to give a simple generalization for distributions which is closely related to Mikusiński operators and convolution quotients of other types **[11, 12, 4, 13]**. The method employed here is an appropriate modification of a general algebraic method **[5, 2, 8]**.

Mappings which commute with convolution are called convolution multipliers here. (Distributions can be characterized as convolution multipliers, Mikusiński operators themselves are convolution multipliers.)

In §1, convolution multipliers from various subsets of $\mathscr{D}$ into $\mathscr{E}$ are discussed. We are primarily concerned with their maximal extensions.

In §2, a module $\mathfrak{M}$ of certain maximal convolution multipliers is constructed and investigated from an algebraic point of view.

In §3, Schwartz distributions are embedded and characterized in $\mathfrak{M}$. For example, we prove that distributions are the only continuous elements of $\mathfrak{M}$. Finally, we show that there are elements in $\mathfrak{M}$ which are not distributions.

To illustrate the appropriateness of our generalizations, we refer to the following facts:

One of the difficulties in working with Schwartz distributions is that only distributions Λ satisfying $\Lambda * \mathscr{D} = \mathscr{D}$ are invertible in $\mathscr{D}'$. Whereas, distibutions Λ satisfying $\Lambda * \mathscr{D} \subset \mathscr{D}$ such that $\Lambda * \mathscr{D}$ has no proper annihilators in $\mathscr{E}$ are invertible in $\mathfrak{M}$. (The heat operator in two dimensions **[1]** seems to be a distribution which is not invertible in $\mathscr{D}'$, but is invertible in $\mathfrak{M}$.)

There are regular Mikusiński operators **[1]** which are not distributions. Whereas, normal Mikusiński operators **[11]** can be embedded in $\mathfrak{M}$.

1. Convolution multipliers and their maximal extensions. Let k be a fixed positive integer, $\mathbf{R}^k$ be the k-dimensional Euclidean space and $\mathbf{C}$ be the field of complex numbers.