PACIFIC JOURNAL OF MATHEMATICS
Vol. 60, No. 2, 1975

UNCONDITIONAL SCHAUDER DECOMPOSITIONS OF NORMED IDEALS OF OPERATORS BETWEEN SOME l_p-SPACES

Y. GORDON

Given a Banach space E, let

$$l(E) = \sup_{F \in \mathscr{F}(E)} \inf_{\{P_i\}} \sup_{N, \pm} \left\| \sum_{i=1}^{N} \pm \sqrt{r(P_i)}\, P_i \right\|$$

where $\mathscr{F}(E)$ denotes the collection of all finite-dimensional subspaces of E, the infimum ranges over all possible sequences of finite-rank operators $P_i: F \to E$ which satisfy the equality $\Sigma P_i(f) = f$ for all $f \in F$, and $r(P)$ denotes the rank of an operator P.

It is shown that there are finite-dimensional spaces with arbitrarily large $l(E)$ values, and infinite-dimensional spaces E with $l(E) = \infty$. The specific examples with $l(E) = \infty$ yield also information on the rapidity of growth of unconditional Schauder decompositions of E into finite-dimensional spaces.

Clearly if E is finite-dimensional

$$l(E) = \inf_{\{P_i\}} \sup_{N, \pm} \left\| \sum_{i=1}^{N} \pm \sqrt{r(P_i)}\, P_i \right\|$$

where the infimum ranges over all sequences $P_i: E \to E$ satisfying $\Sigma_{i \geq 1} P_i(x) = x$ for all $x \in E$.

It is also obvious from the definition that the value $l(E)$ is not greater than the local unconditional constant $\chi_u(E)$ introduced in [3] which is defined similarly, the only difference being that for $\chi_u(E)$ only sequences $\{P_i\}$ with $r(P_i) = 1$ for all i are considered. Spaces E with finite $\chi_u(E)$ were called in [3] spaces with local unconditional structure. If E is complemented in a space with an unconditional basis then clearly $\chi_u(E) < \infty$.

Besides this generalization the result stated above answers a question of Professor H. P. Rosenthal by providing examples of spaces which do not have unconditional Schauder decompositions into finite-dimensional spaces all of the same dimension p, for any $p = 1, 2, 3, \cdots$; spaces E with $l(E) = \infty$ clearly cannot have such decompositions.

Specifically it is shown in section 2 that if E is the space of operators on l_2 equipped with any ideal norm α, then $l(E) = \infty$ unless α is