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SUBCONTINUITY FOR MULTIFUNCTIONS

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The concept of subcontinuity is extended to multifunctions. This notion is then used to obtain a number of results on multifunctions with closed graphs and to develop criteria under which a multifunction is upper semicontinuous.

1. Introduction.

In [1] R. V. Fuller introduced the concept of subcontinuous function and used it to obtain conditions implying continuity as well as some comparisons to compactness preserving functions and some results on functions with closed graphs. In [1] Fuller stated that he thought that his results would hold for multifunctions. The purpose of this paper is to show that such is the case. Indeed the definition of subcontinuity can be extended and conditions implying upper semi-continuity for multifunctions are derived, including a generalization of a result of Muenzenberger and Smithson [4].

By a *multifunction* we mean a correspondence $F: X \to Y$ on a set X into a set Y such that $F(x)$ is a nonempty subset of Y for each $x \in X$. If F is a multifunction, then the *graph of* F is the subset $\{(x, y): x \in X, y \in F(x)\}$ of $X \times Y$. We denote the graph of F by $G(F)$. Then a multifunction F has a *closed graph* if $G(F)$ is a closed subset of $X \times Y$. Whereas F is a *closed* multifunction if $F(A)$ is closed in Y for all closed sets $A \subset X$. Further, F is *upper semicontinuous* (u.s.c.) if and only if for each closed set $B \subset Y$, $F^{-1}(B) = \{x: F(x) \cap B \neq \emptyset\}$ is closed. This is equivalent to saying that F^{-1} (where $F^{-1}(y) = \{x: y \in F(x)\}$) is a closed multifunction on $F(X) = \bigcup \{F(x): x \in X\}$.

If $A \subset X$, then A^- denotes the closure of A. Other notation used is that of Kelley [3].

2. Subcontinuous multifunctions.

Using Fuller's definition as a model we obtain the following definition.

DEFINITION. A multifunction $F: X \to Y$ is *subcontinuous* if and only if whenever $\{x_\alpha, \alpha \in D\}$ is a convergent net in X and $\{y_\alpha, \alpha \in D\}$ is a net in $F(X)$ with $y_\alpha \in F(x_\alpha)$ then $\{y_\alpha, \alpha \in D\}$ has a convergent subnet.

Next we say that F is *inversely subcontinuous* in case F^{-1} is subcontinuous. Note that since F and F^{-1} are both multifunctions we did not need a separate definition in terms of nets for inverse subcontinuity. Also, if F is single valued then the present definitions