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THE POLYNOMIAL HULLS OF CERTAIN SUBSETS OF C^2

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Summary. Fix $\alpha \leqq 0$. We will describe the polynomial hulls of compact subsets of C^2 that are invariant under the transformations T_θ, $0 \leqq \theta \leqq 2\pi$, defined by

$$T_\theta(z,w) = (e^{i\theta}z, e^{i\alpha\theta}w), \qquad (z,w) \in C^2. \tag{0.1}$$

1. Introduction. Let X be a compact subset of C^2. In [10], J. Wermer describes the polynomial hull $\hat{X}$ of X, for a certain class of sets X which are invariant under the one-parameter group of transformations

$$(z, w) \longrightarrow (e^{i\theta}z, e^{-i\theta}w), \qquad 0 \leqq \theta \leqq 2\pi. \tag{1.1}$$

He develops an idea for introducing analytic structure in $\hat{X}\backslash X$, and he shows in particular that every point of $\hat{X}\backslash X$ lies on an analytic disc in $\hat{X}$.

Our aim is to combine Wermer's ideas with some elementary results in potential theory, in order to describe the polynomial hull $\hat{X}$ of an arbitrary compact subset X of C^2 invariant under the group (1.1), or more generally under the transformations defined by (0.1).

If the number α is irrational, then the transformation group defined by (0.1) is dense in the two-parameter group of transformations

$$(z, w) \longrightarrow (e^{i\theta}z, e^{i\varphi}w), \qquad 0 \leqq \theta, \varphi \leqq 2\pi. \tag{1.2}$$

If X is invariant under the group (0, 1), then X is also invariant under the groups (1.2). Such sets are said to be *circled*. The description of the polynomial hull of compact circled sets is classical ([6], [2], [3], [4, §III. 3], [7, §2.4], [9, §14]). We may confine our attention to the case in which α is rational.

Consider next the case $\alpha = 0$. The transformations (0.1) then assume the form

$$(z, w) \longrightarrow (e^{i\theta}z, w), \qquad 0 \leqq \theta \leqq 2\pi. \tag{1.3}$$

If X is invariant under the transformations (1.3), its polynomial hull can be described as follows [9]. Let J be the projection of X into the w-plane, and define

$$r(w) = \sup\{|z| : (z, w) \in X\}, \qquad w \in J.$$

Define a function R on the polynomial hull $\hat{J}$ of J by requiring that $\log R$ be the lower envelope of the family of functions u which are