PACIFIC JOURNAL OF MATHEMATICS
Vol. 62, No. 1, 1976

# COMPOSITION OPERATORS ON $H^p(A)$

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**The space $H^p(A)$ is a generalization of the Hardy space $H^p$ for functions analytic on an annulus $A$. This paper shows that composition operators are bounded operators on $H^p(A)$ and obtains an upper bound on the norm of the operator. The space $H^2(A)$ is given a Hilbert space structure and those composition operators that are in the Hilbert–Schmidt class of operators on $H^2(A)$ are characterized in terms of integral properties of the inducing function.**

**1. Introduction.** If $H$ is a space of functions analytic on a region $R$, and if $\varphi$ is an analytic map of $R$ into itself, the composition operator $C_\varphi$ on the space $H$ is defined by $C_\varphi(f) = f \circ \varphi$. In recent articles composition operators have been studied on various function spaces including the Hardy space $H^p$ and the Bergman space $A^2$. See, for example, (2), (4), (7), and (8). In all these spaces the underlying region of analyticity was the unit disk. In this paper we show that composition operators form bounded operators on the space $H^p(A)$, $0 < p < \infty$, a generalization of the Hardy space which consists of functions analytic on an annulus. In addition, a characterization of those composition operators which form Hilbert–Schmidt operators on the Hilbert space $H^2(A)$ is derived.

**2. Boundedness of composition operators.** A well-known generalization of $H^p$, $0 < p < \infty$, was given by Rudin in (5). For $r$ in $(0, 1)$ he considered the linear space of functions $f$ analytic on $A = (z: r < |z| < 1/r)$ with the property that $|f|^p$ has a harmonic majorant on $A$. He showed that for $p \geqq 1$ this space is a Banach space under the norm $\|f\|_p = (u(1))^{1/p}$, where $u$ is the least harmonic majorant of $|f|^p$. When $0 < p < 1$, this space is an $F$ space (i.e., a complete translation invariant metric space) when given the metric $d(f, g) = \|f - g\|_p^p$.

Another generalization for $p \geqq 1$ was introduced by Sarason in (6) as the space of functions $f$ analytic on $A$ with bounded integral means $M_p(f, r)$. He showed that such functions have nontangential limits almost everywhere on the boundary and that the space is a Banach space under the norm

$$\|f\|_p = (M_p^p(f, r) + M_p^p(f, 1/r))^{1/p}.$$