PACIFIC JOURNAL OF MATHEMATICS
Vol. 62, No. 1, 1976

SOME MAPPINGS WHICH DO NOT ADMIT AN AVERAGING OPERATOR

JOHN WARREN BAKER AND R. C. LACHER

The problem of determining for spaces X and Y necessary and sufficient conditions such that there exists a map ϕ of X onto Y which does not admit an averaging operator is considered. This corresponds to identifying the uncomplemented closed selfadjoint subalgebras of $C(X)$ which contain 1_X. Mappings ϕ of X onto Y are constructed which do not admit averaging operators, for example, when X is any uncountable compact metric space and Y is any countable product of intervals. Also, X can be any space containing an open set homeomorphic to a Banach space and $Y = X$. These results generalize earlier work by D. Amir and S. Ditor.

If ϕ is a mapping of X onto Y, the induced operator ϕ^0 from $C(Y)$ to $C(X)$ that takes $f \in C(Y)$ to $f \circ \phi \in C(X)$ is a multiplicative isometric isomorphism. In case ϕ is a quotient map (e.g., if X and Y are compact Hausdorff spaces) then $\phi^0(C(Y))$ consists of all functions in $C(X)$ which are constant on each point inverse of ϕ. We say ϕ *admits an averaging operator* if there is a projection of $C(X)$ onto $\phi^0(C(Y))$. It is easily seen that ϕ admits an averaging operator if and only if there exists a bounded linear operator u from $C(X)$ into $C(Y)$ such that $u\phi^0(f) = f$ for each $f \in C(Y)$ (see **[12]**, Cor. 3.2), and in this case u is called an *averaging operator* for ϕ.

Following the appearance of the monograph by A. Pelczynski on averaging and extension operators **[12]**, there has been much interest in the study of averaging operators (e.g., see **[2]**, **[3]**, **[4]**, **[5]**, **[6]**, **[15]**). A central problem in this study, known as the complemented subalgebra problem, is to determine necessary and sufficient conditions for a map ϕ from a compact Hausdorff space X onto a compact Hausdorff space Y to admit an averaging operator. Strong necessary conditions have been established in **[5]**. (Also, see **[2]** and **[3]**.) Two closely related problems are to determine for compact Hausdorff spaces X and Y necessary and sufficient conditions that there exists a map ϕ of X onto Y which (1. admits; 2. does not admit) an averaging operator. Since this corresponds to determining the complemented and uncomplemented closed selfadjoint subalgebras of $C(X)$ which contain 1_X by Stone's Theorem **[14**, p. 122], results of this type yield information about the structure of $C(X)$.

In 1968, S. Ditor established that there is a map ϕ of $[0, 1]$ onto itself