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SUBGROUPS OF FINITE INDEX IN PROFINITE GROUPS

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A profinite group is called strongly complete if every subgroup of finite index is open and of type (AF) if it has only finitely many subgroups of any fixed index. In this paper it is shown that a topologically finitely generated abelian by pro-nilpotent profinite group is strongly complete, and that a pro-solvable profinite group is strongly complete if is of type (AF).

In **[6]** it is shown that every strongly complete group is of type (AF). The proof of the converse of this result for pro-solvable groups rests on two facts. The first is that any finite quotient of a pro-solvable profinite group is solvable. The second is that any measurable subgroup of finite index in a profinite group is open. In particular, any verbal subgroup of finite index is open.

Lazard (**[4]**, Chap. III, 3.) has shown that every pro-p-group which admits a p-adic analytic structure is strongly complete and Serre **[8]** has extended this result to all topologically finitely generated pro-p-groups. The proof given here that topologically finitely generated abelian by pro-nilpotent groups are strongly complete combines Serre's technique with a similar technique used to prove the result for metabelian profinite groups. All these proofs rest on the fact that the commutator subgroup of the group in question is closed.

The paper is concluded with some examples and a remark on the relationship of the problems considered here with some problems in the theory of finite groups.

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1. Abstract properties of profinite groups.

Let G be a profinite group. We will denote the order of G (cf. **[7]**) by $o(G)$. If x is an element of G we define the exponent of x to be the order of the closed subgroup of G generated by x.

Let $\hat{G}$ be the completion of the abstract group G for the topology having as neighborhood basis of the identity the subgroups of finite index. There is a natural homomorphism $G \to \hat{G}$. Since $\hat{G}$ is universal for continuous homomorphisms of the discrete group G to profinite