PACIFIC JOURNAL OF MATHEMATICS
Vol. 62, No. 2, 1976

ON THE STRUCTURE OF ALGEBRAIC ALGEBRAS

THOMAS J. LAFFEY

Let A be an infinite dimensional (associative) algebra over a field F. It is shown that A has an infinite dimensional commutative subalgebra C of *one* of the following types:

(a) C is generated by one element
(b) $C^2 = \{0\}$
(c) $C^3 = \{0\}$ and C is an ideal of an ideal of A
(d) C is generated by mutually orthogonal idempotents
(e) C is a field.

A necessary and sufficient condition (in terms of the quadratic forms over F) is obtained for the validity of the statement: Every infinite dimensional nil algebra over F has an infinite dimensional subalgebra B with $B^2 = \{0\}$. An ideal $y(A)$ of an algebra A (analogous to the F.C. subgroup in group theory) is defined and several properties of it are obtained.

In §1 of this paper we give a list of notation and definitions. In §2 we prove some preliminary results on rings and algebras. In §3 we obtain several properties of an ideal $y(A)$ of an algebra A (defined in §1). The ideal $y(A)$ is an algebra-theoretic analogue of the F.C. (finite conjugate) subgroup in group theory. The main result of this paper is the Main Theorem, proved in §4, which states:

MAIN THEOREM (4.1). *Let A be an infinite dimensional (associative) algebraic algebra over a field F and assume*

(a) *A contains no infinite set of mutually orthogonal idempotents.*
(b) *A contains no infinite dimensional subalgebra B with $B^2 = 0$*
(c) *$y(A)$ is finite dimensional.*

Then $A = M \oplus N$ where M is finite dimensional and N is the direct sum of finitely many algebraic division algebras.

An immediate consequence of the Main Theorem is the fact, proved in **[5]**, that every infinite dimensional associative algebra over a field has an infinite dimensional commutative subalgebra.

Several results in this paper deal with algebraic algebras which have no infinite dimensional subalgebras with square zero. This condition restricts the structure of an algebraic algebra considerably. For example, if A is a nil algebra over a field and A has no infinite dimensional subalgebra C with $C^2 = 0$, then A is locally finite and A is even necessarily finite dimensional for a large class of fields.