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# WEAK TYPE MULTIPLIERS FOR HANKEL TRANSFORMS

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**The main result of this paper is that weak type multiplier theorems for Jacobi expansions yield weak type multiplier theorems for Hankel transforms.**

**In recent papers the authors studied multiplier theorems for ultraspherical and Jacobi expansions. An interesting paper of Igari suggested a new approach to multiplier theorems that used asymptotics instead of the elaborate machinery used earlier.**

**This paper extends the method of Igari to give the first weak type multiplier theorem for Hankel transforms. This extension is important in itself and because this method together with the authors' results for Jacobi multipliers will generalize to the "radial" functions associated with the other compact symmetric spaces.**

Let $\{P_n^{(\alpha,\beta)}(x)\}$ be the Jacobi polynomials with indices $(\alpha, \beta)$. The functions $\{P_n^{(\alpha,\beta)}(\cos\theta)\}$ are orthogonal with respect to the measure $d\mu(\theta) = (\sin\theta/2)^{2\alpha+1}(\cos\theta/2)^{2\beta+1}d\theta$. For measurable $f$ on $[0, \pi]$ define

$$||f||_p = \left\{\int_0^\pi |f(\theta)|^p d\mu(\theta)\right\}^{1/p}$$

and

$$f\hat{\ }(n) = \int_0^\pi f(\theta)P_n^{(\alpha,\beta)}(\cos\theta)d\mu(\theta)$$

so that if

$$h_n^{-1} = \int_0^\pi [P_n^{(\alpha,\beta)}(\cos\theta)]^2 d\mu(\theta)\ ,$$

then

$$f(\theta) \sim \sum_{n=0}^\infty f\hat{\ }(n)h_n P_n^{(\alpha,\beta)}(\cos\theta)$$

where equality holds at least for finite series.

The multiplier transformation defined by the function $\phi(x)$ is denoted by $T_\phi$, where at least formally,

$$T_\phi f(\theta) \sim \sum_0^\infty \phi(n)\hat{f}(n)h_n P_n^{(\alpha,\beta)}(\cos\theta)\ .$$

The operator $T_\phi$ is said to be of strong type $p$, $1 < p < \infty$ if